Gravitational binding energy in general relativity

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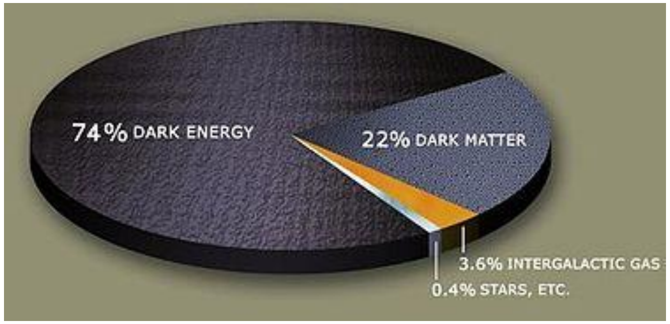
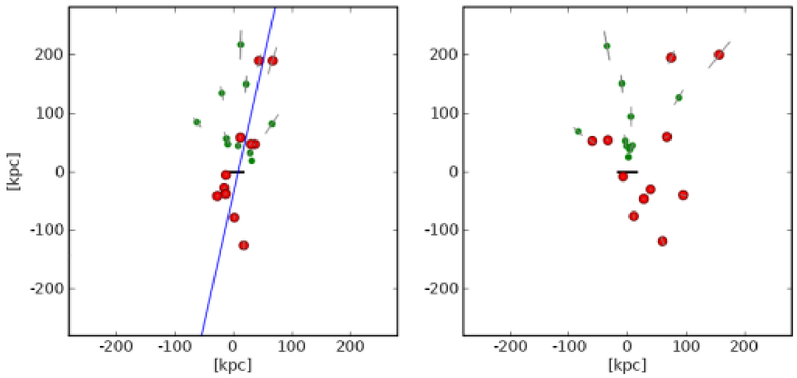
The ΛCDM-model

- Cosmological principle (homogeneity and isotropy → RW-metric)
- The Universe has been in a very hot and dense state (→ Big-Bang)
- Cosmic expansion is accelerated ($\ddot{R} \neq 0$)
- WMAP & COBE → $T = 2.725 \pm 0.002°K$
- $k = 0$
- $\Omega_b = 0.046, \Omega_d = 0.23, \Omega_\Lambda = 0.73$



Dubious assumptions & problems

- The Universe is not homogeneous (on small scales)
- Initial singularity
- $\Delta T = f(k)$
- The disc of satellites
- Missing satellite problem
- Constant vacuum energy density



Einstein field equations and energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} R \cdot g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$T_{\mu\nu} = \left(\rho + \frac{P^2}{c^2}\right) u_\mu u_\nu - g_{\mu\nu} P$$

$T_{\mu\nu}$ contains all energy forms acting as source of gravity

BUT: Not binding energy!!!

How does binding energy act?

Fischer (1993):

$$\left(T_{\mu\nu}\right)_b = -C\frac{\rho}{\Gamma}g_{\mu\nu}$$

Formally it has the same action as the term connected with the action of vacuum energy denstity

$$\Rightarrow \rho_{eff} = \rho\left(1 - C\frac{1}{\Gamma}\right)$$

Effective mass density is reduced!

How does binding energy act?

→ New expansion equations

$$\frac{1}{R^2}+\left(\frac{\dot{R}}{R}\right)^2+2\frac{\ddot{R}}{R}=\frac{8\pi G}{c^4}\frac{C\rho}{R}$$

$$-3\left(\frac{1}{R^2}+\left(\frac{\dot{R}}{R}\right)^2\right)=-\frac{8\pi G}{c^4}\left(1-\frac{C}{R}\right)\rho$$

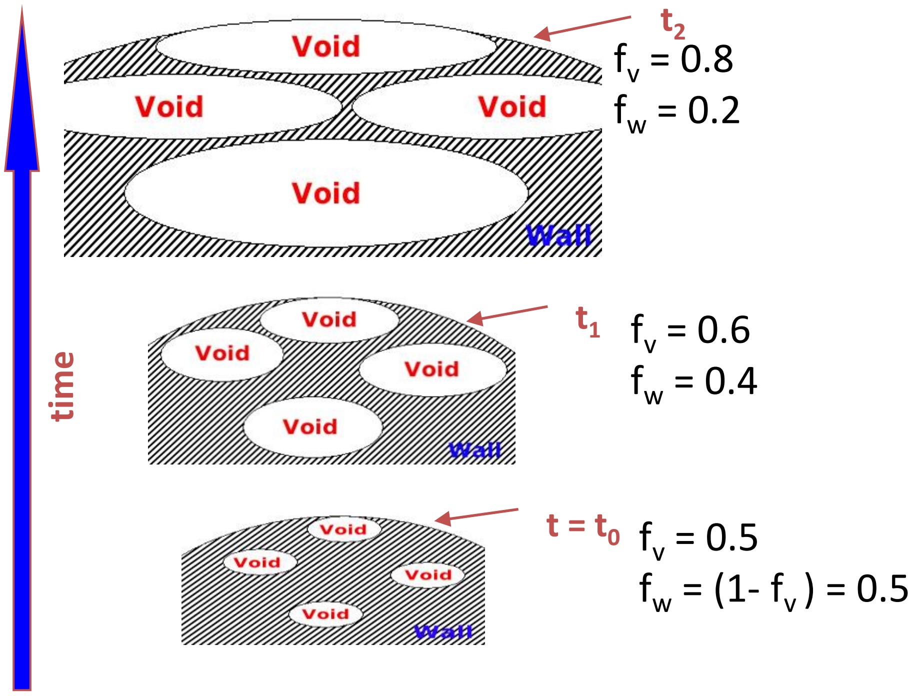
$$\frac{\ddot{R}}{R}=\frac{4\pi G}{3c^4}\left(\frac{R_0}{R}-1\right)\rho$$

How does binding energy act?

Wiltshire (2007):

2-phase-universe

Volume fill-factors f_v, f_w



How does binding energy act?

$$\bar{\rho}_2 = \rho_v f_v + \rho_w f_w = \rho_v f_v + \rho_w (1 - f_v)$$

Reduced density → $\boxed{\bar{\rho}_2(f_v \geq 0.57) = \bar{\rho}_2 \left(1 - \frac{1-2\bar{q}_2}{2(\bar{q}_2+1)}\right)}$

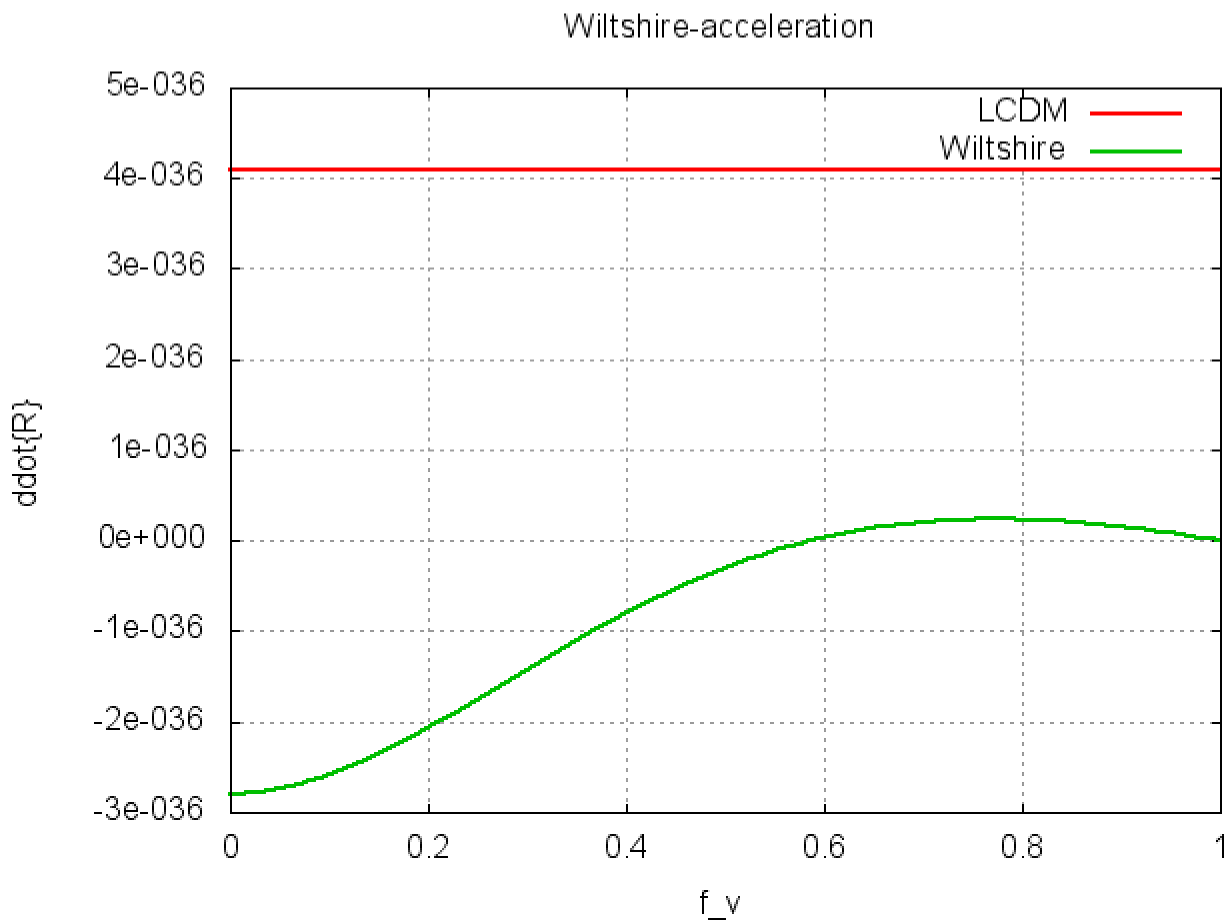
Acceleration-parameter $q := -\frac{\ddot{R}R}{\dot{R}^2}$

$$\rightarrow \bar{q}_2(f_v) = -\frac{(1-f_v)(8{f_v}^3+39{f_v}^2-12f_v-8)}{4+f_v+4{f_v}^2}$$

For $f_v \geq 0.57 \Rightarrow \bar{q}_2(f_v) < 0$ → Accelerated expansion

$$\bar{q}_2(f_v)(-{H_0}^2) = -\bar{q}_2(f_v)\left(\frac{\dot{R}}{R}\right)^2 = \frac{\ddot{R}}{R}$$

How does binding energy act?



How does binding energy act?

Caution:

- Wiltshire starts out from scalar differential equations (Friedmann-Eqns)
- Correct treatment would imply calculation of backreaction-termns starting from lon-linear 2nd-order differential equations coming from tensor formalism of the GRT field equations.

How does binding energy act?

Fahr & Sokaliwska 2011/2012:

Two-point correlation function $\xi(l) = \xi_0 \left(\frac{l_0}{l}\right)^{\alpha}$

- l_0 inner scale, typical for galaxies
- α power index $\simeq 1.8$

Local potential energy $\rho(l) = \rho_0 \left(\frac{l_0}{l}\right)^{\alpha}$

To conserve initial mass $\rho_0 = \frac{3-\alpha}{3} \bar{\rho} \left(\frac{l_m}{l_0}\right)^{\alpha}$

(l_m = outer integration scale)

How does binding energy act?

Potential energy is then given by

$$\varepsilon_{pot} = \frac{16\pi^2(3-\alpha)}{9(5-2\alpha)} G\bar{\rho}^2 l^5$$

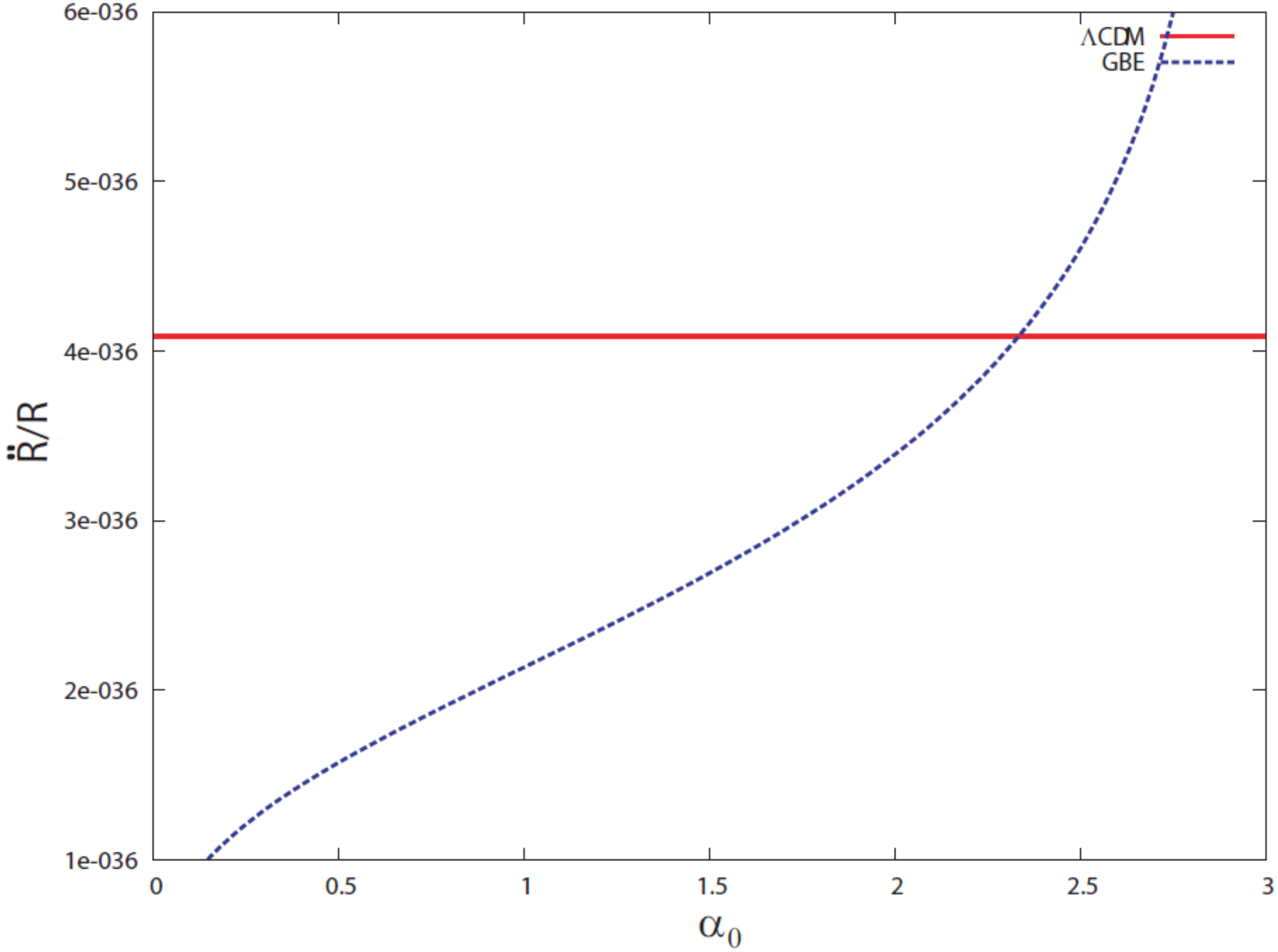
Reduced density →

$$\rho_{eff} = \bar{\rho}\left(1 - \frac{\rho_\alpha}{\bar{\rho}}\right) = \bar{\rho}\left(1 - \frac{4\pi(3-\alpha)}{3(5-2\alpha)}\frac{G}{c^2}\bar{\rho}{l_m}^2\right)$$

$$\left(\frac{\ddot{R}}{R}\right)_\alpha = 4\pi\rho\left[\left(\frac{\alpha(\alpha_0-3)}{\alpha_0(\alpha-3)}\right)^{2/3}\frac{(3-\alpha)^2\alpha_0}{3(3-\alpha_0)(5-2\alpha)^2}\right]$$

$$l_m = Rexp\left[-\int_{\alpha_0}^{\alpha}\frac{d\alpha}{(3-\alpha)\alpha}\right] = R\left(\frac{(\alpha_0-3)\alpha}{(\alpha-3)\alpha_0}\right)^{-1/3}$$

How does binding energy act?



Summary

- Fischer: $$\rho_{eff} = \rho\left(1 - C\,\frac{1}{\Gamma}\right)$$
- Wiltshire: $$\bar{\rho}_2(f_v \geq 0.57) = \bar{\rho}_2\left(1 - \frac{1-2\bar{q}_2}{2(\bar{q}_2+1)}\right)$$
- Fahr: $$\rho_{eff} = \bar{\rho}\left(1 - \frac{4\pi(3-\alpha)}{3(5-2\alpha)}\frac{G}{c^2}\bar{\rho}{l_m}^2\right)$$

Short outlook

- Include ρ_α into field equations
- Derive new Expansion equations
- Integrate numerically → R(t)
- Fit to SNIa-data

Thank You!

Proper density by metrical distortion

- Λ should be coupled to matter density
- „Matter density“ is connected with the spacetime geometry
- $\rho = \frac{m}{V}$ is in fact problematic in curved space!
- Metrical distortion of unit volumnes
- → Reduction of proper density

$$\rho_{eff} = \rho\left(1 - \left(\frac{\rho}{\rho_c}\right)^{1/3}\right)$$

(Fahr & Heyl, 2007 a,b)