Collection of basic equations on SZ

Frank Bertoldi (06.12.02)

Following the notation of Diego et al. (2002; MNRAS 331, 556) the SZ comptonization parameter is

$$y_c = \frac{\sigma_T k_B}{m_e c^2} \int T n_e \, dl \tag{1}$$

and the CMB distortion

$$\frac{\delta T}{T_{\rm CMB}} = g(x) \; y_c \tag{2}$$

where the dimensionless frequency

$$x = \frac{h\nu}{kT_{\rm CMB}} = \frac{\nu}{56.8{\rm GHz}} = \frac{2.6 \; \nu}{150{\rm GHz}} = \frac{6.2 \; \nu}{353{\rm GHz}} \tag{3}$$

and the spectral shape factor

$$g(x) = x \; \coth(x/2) - 4 = x \; \frac{e^x + 1}{e^x - 1} - 4 \; = \; \begin{cases} -1 & : \; 150 \text{ GHz} \\ 2.2 & : \; 353 \text{ GHz} \end{cases} \tag{4}$$

The change in intensity induced by the SZE is given by

$$\frac{\delta I(\nu, \vec{\theta})}{I_0} = f(x) \; y_c \tag{5}$$

where

$$I_0 = 2 \; \frac{(k_B T_{\rm CMB})^3}{(h_p c)^2} = 2.7 \times 10^{11} \text{ mJy str}^{-1} \tag{6}$$

(I don't understand properly where the str comes from. Note: 1 mJy = 1.0×10^{-26} erg s^{-1} Hz^{-1} cm^{-2}) and

$$f(x) = g(x) \cdot h(x) \; = \; \begin{cases} -4.0 & : \; 150 \text{ GHz} \\ 6.65 & : \; 353 \text{ GHz} \end{cases} \tag{7}$$

is the spectral shape, with

$$h(x) = \frac{x^4 e^x}{(e^x - 1)^2} \; = \; \begin{cases} 4.0 & : \; 150 \text{ GHz} \\ 3.07 & : \; 353 \text{ GHz} \end{cases} \tag{8}$$

The total SZ flux density from the cluster is given by

$$S_{\rm SZE}(\nu) = \int_{cluster} \delta I(\nu, \vec{\theta}) \; d\Omega \tag{9}$$

where the integral is performed over the solid angle subtended by the cluster. Assuming that the cluster is isothermal the integral can be reduced to $\propto \int d\Omega \int n_e(\vec{\theta}) d\ell$ which can be transformed into $D_a^{-2}(z) \int dV n_e(\vec{\theta}) = D_a^{-2}(z) \frac{M_{gas}}{m_p} = D_a^{-2}(z) \frac{f_b}{m_p} M$. M, f_b and m_p are the total mass, baryon fraction and the proton mass respectively. We assumed that the gas is only composed of ionized Hydrogen. Then:

$$S_{\rm SZE} = 0.80 \; \frac{T}{10^7 {\rm K}} \; \frac{M}{10^{14} M_\odot} \; \frac{f(x)}{4} \; \frac{f_b}{0.06} \; \left(\frac{1072 \; {\rm Mpc}}{D_a} \right)^2 \; {\rm mJy} \tag{10}$$

The angular diameter distance is given (which cosmology?) by

$$D_a = \frac{c}{H_0 q_0^2} \frac{(q_0 z + (q_0 - 1)(\sqrt{1 + 2q_0 z} - 1))}{(1+z)^2} = \frac{c}{H_0} \frac{z}{(1+z)^2} \quad \text{for} \quad q_0 = 1 \tag{11}$$

Assuming $H_0 = 70$ km s^{-1}Mpc^{-1} and $q_0 = 1$, D_a reaches a maximum of 1072 Mpc at $z = 1$. The intensity variation $\delta I(\nu)$ can be related to the temperature variation δT through

$$\frac{\delta I(\nu)}{I_0} \; = \; h(x) \; \frac{\delta T}{T_{\rm CMB}} \tag{12}$$

With a beam FWHM of 45 arcsec at 150 GHz the APEX beam size $\Omega_{150} \sim 5 \times 10^{-8}$ str, so that for a point source the observed SZE flux density

$$S_{\rm SZE} \; = \; \Omega_{150} \; \delta I (150 {\rm GHz}) \; = 0.20 \; \frac{\delta T}{10 \mu{\rm K}} \; {\rm mJy} \tag{13}$$

If we map the CMB to an RMS of 10μK per beam, a 5σ point source detection limit would correspond to a cluster SZ flux density of of 1 mJy.

Assuming a halo to be isothermal, the total mass of a halo is related to the gas temperature by

$$\frac{T}{T_8} = \left(\frac{M}{M_8} \right)^{2/3} \tag{14}$$

where

$$M_8 = 1.8 \cdot 10^{14} h^{-1} (\Omega_m / 0.3) M_\odot$$

is the mass in a $8h^{-1}$ Mpc sphere of mean density today, which is roughly the mass scale of clusters, and $T_8(z)$ is the corresponding temperature of a halo with mass M_8 at redshift z (from Zhang et al. 2002, ApJ 577, 555). Comparing the cluster temperature function as inferred from simulations with the Press-Schechter formalism, Pen (1998, ApJ 498, 60) found that

$$T_8 = 4.9 \; (1+z) \; \Omega_m^{2/3} \; \Omega(z)^{0.283} \; {\rm keV}$$

for a Λ CDM universe (note: 1 keV $\equiv$ 11.6 Mio K). Explicitly,

$$T = 2.71 \cdot 10^7 \; \frac{1+z}{2} \left(\frac{h}{0.7}\right)^{2/3} \Omega^{0.283} \left(\frac{M}{10^{14} M_\odot}\right)^{2/3} \; \mathrm{K} \tag{15}$$

Alternatively, Diego et al. (their eq.8) find

$$T = 2.0 \cdot 10^7 \; \frac{1+z}{2} \; \left(\frac{M}{10^{14} M_\odot}\right)^{0.75} \; \mathrm{K} \tag{16}$$

for a flat ($\Omega_\Lambda = 0.7$) CMD universe with $\sigma_8 = 0.8$ and $\Omega_m = 0.3$ (this should also scale with h and Ω like in expression (15)?). For an open $\Omega_\Lambda = 0$ CDM cosmology they find a similar expression with $T \propto M^{0.8}$.

Adopting the Pen et al. temperature (15), the integrated flux density (10) of a cluster is

$$S_{\mathrm{SZE}} = 2.15 \; \left(\frac{M}{10^{14} M_\odot}\right)^{5/3} \; \frac{f(x)}{4} \frac{f_b}{0.06} \; z^{-2} \left(\frac{1+z}{2}\right)^5 \left(\frac{h}{0.7}\right)^{2/3} \Omega^{0.283} \; \mathrm{mJy} \tag{17}$$

Using the temperature (16) suggested by Diego et al. would yield a somewhat smaller flux density, $S_{\mathrm{SZE}} = 1.59 M_{14}^{1.75}$ mJy.

If we aim at a point source 5σ detection limit of 50 μK equivalent 1.0 mJy, this would correspond to a cluster mass of about $7 \cdot 10^{13} M_\odot$. If the clusters are slightly resolved, the mass detection limit will be somewhat higher.