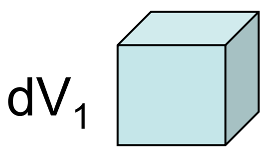
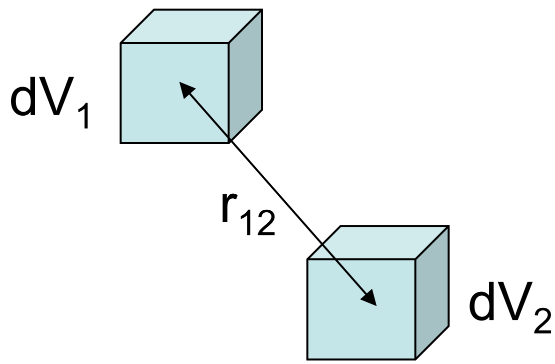
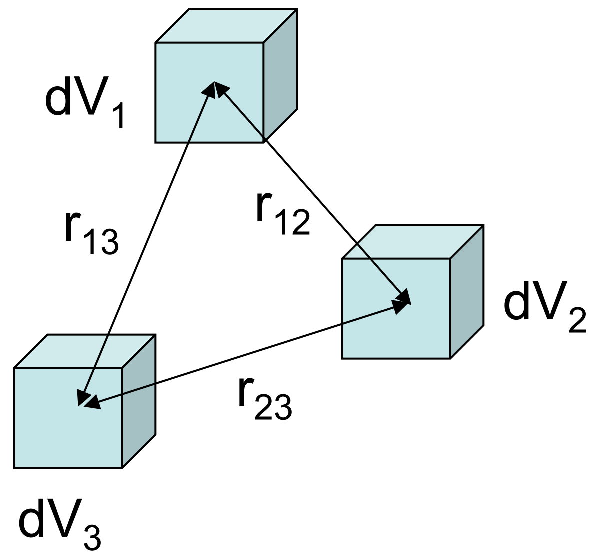
Correlation functions

Consider a stationary point process with mean density n and write the probability of finding N points within N infinitesimal volume elements







$$dP_1 = n\, dV_1$$

$$dP_{12} = n^2\,(1+\xi(r_{12}))\, dV_1\, dV_2$$

$$dP_{123} = n^3\,(1+\xi(r_{12})+ \xi(r_{13})+ \xi(r_{23})+\zeta(r_{12}, r_{13}, r_{23}))\, dV_1\, dV_2\, dV_3$$

Power spectra

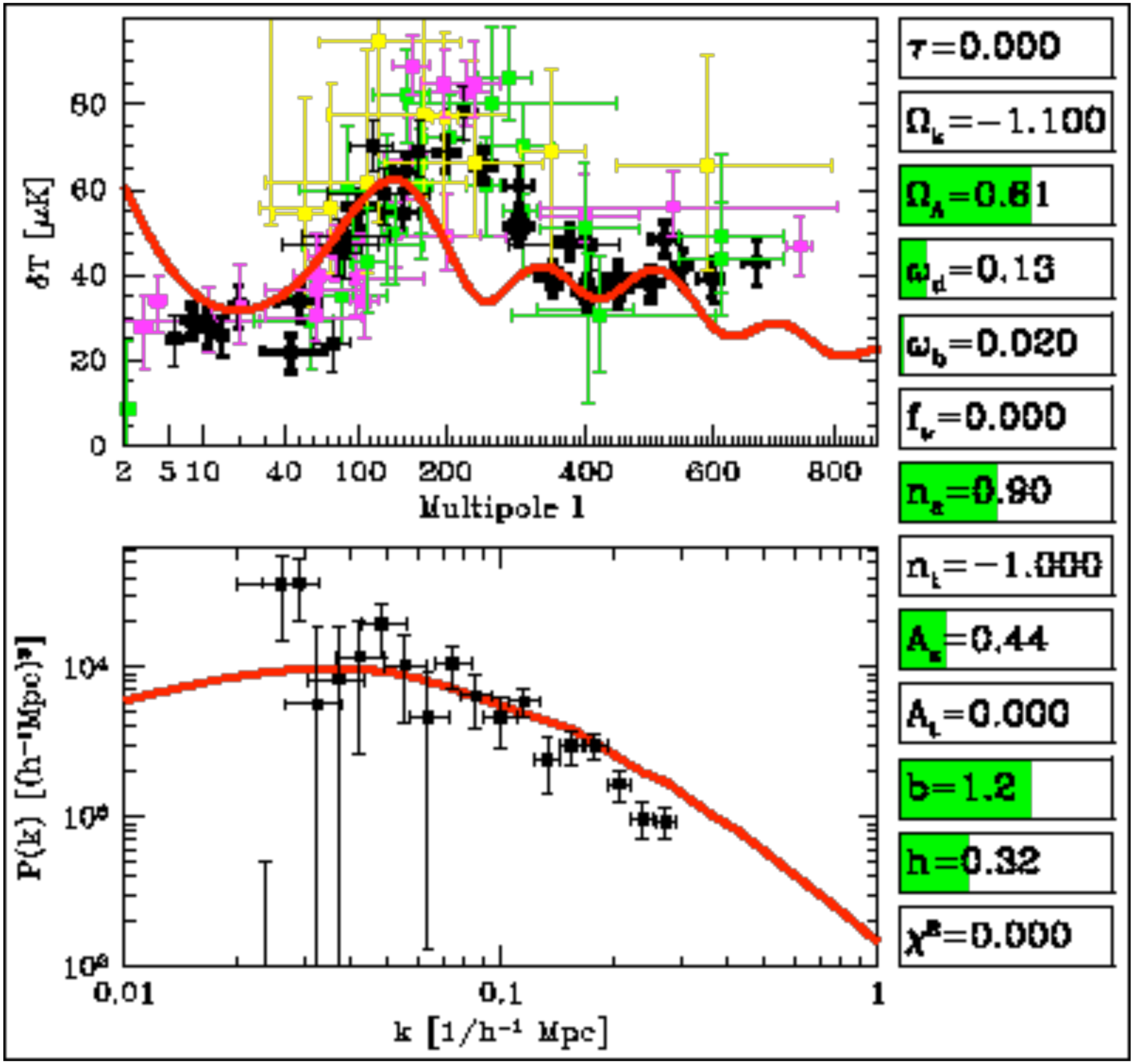
N-spectrum defined via the expectation value of the product of N+1 Fourier transforms of the overdensity field

$$\langle \tilde{\delta}(\vec{k})\,\tilde{\delta}(\vec{q}) \rangle = (2\pi)^3 P(k)\,\delta_D(\vec{k}+\vec{q})$$

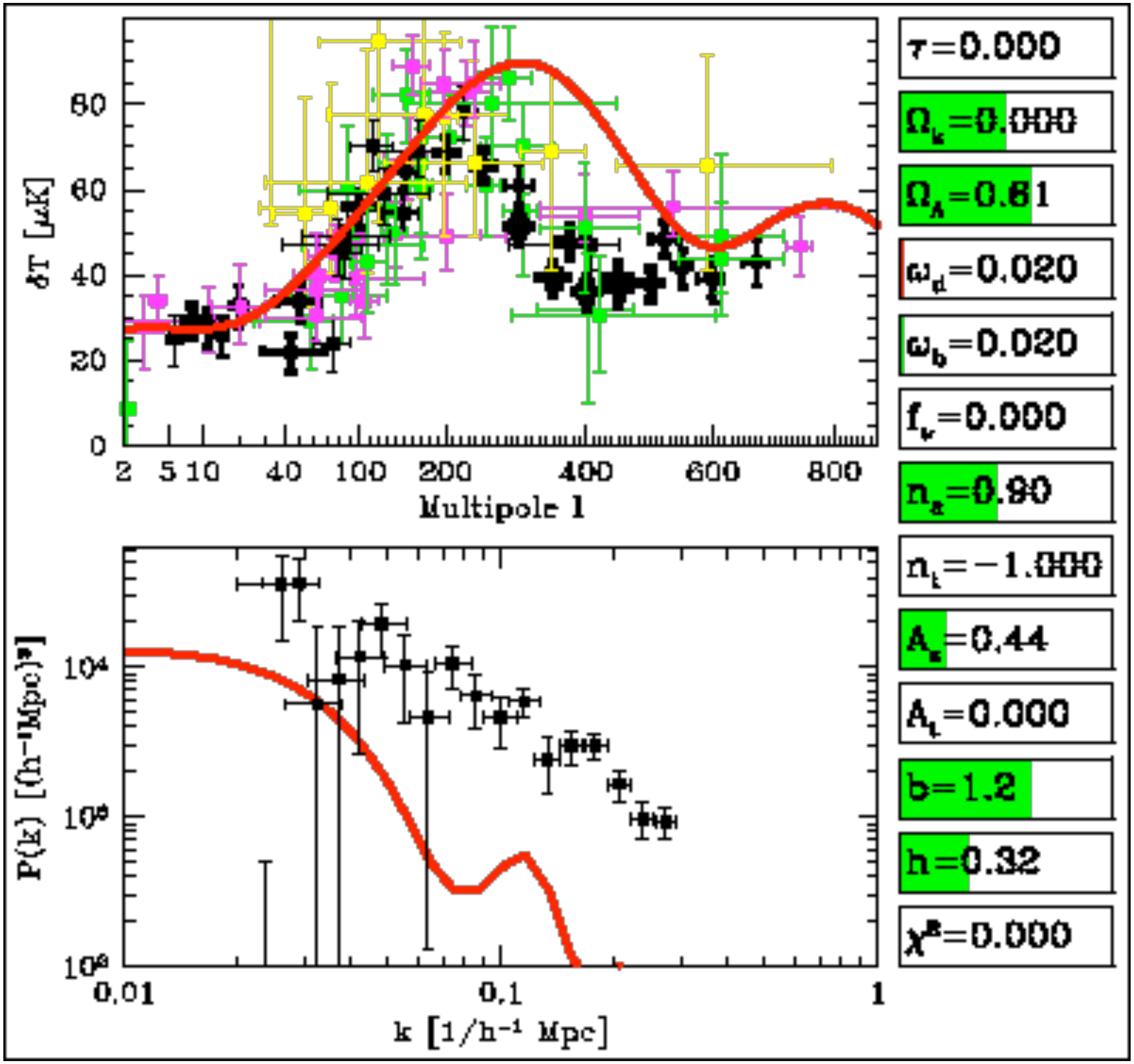
$$\langle \tilde{\delta}(\vec{k})\,\tilde{\delta}(\vec{q})\,\tilde{\delta}(\vec{p}) \rangle = (2\pi)^3 B(k,q,p)\,\delta_D(\vec{k}+\vec{q}+\vec{p})$$

Wiener - Khintchine theorem:

$$\xi(r) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k)\, j_0(kr)\,dk$$



δT [μK]
Multipole l
P(k) [(h⁻¹Mpc)³]
k [1/h⁻¹ Mpc]
τ=0.000
Ω_k=−1.100
Ω_Λ=0.61
ω_d=0.13
ω_b=0.020
f_ν=0.000
n_s=0.90
n_t=−1.000
A_s=0.44
A_t=0.000
b=1.2
h=0.32
χ²=0.000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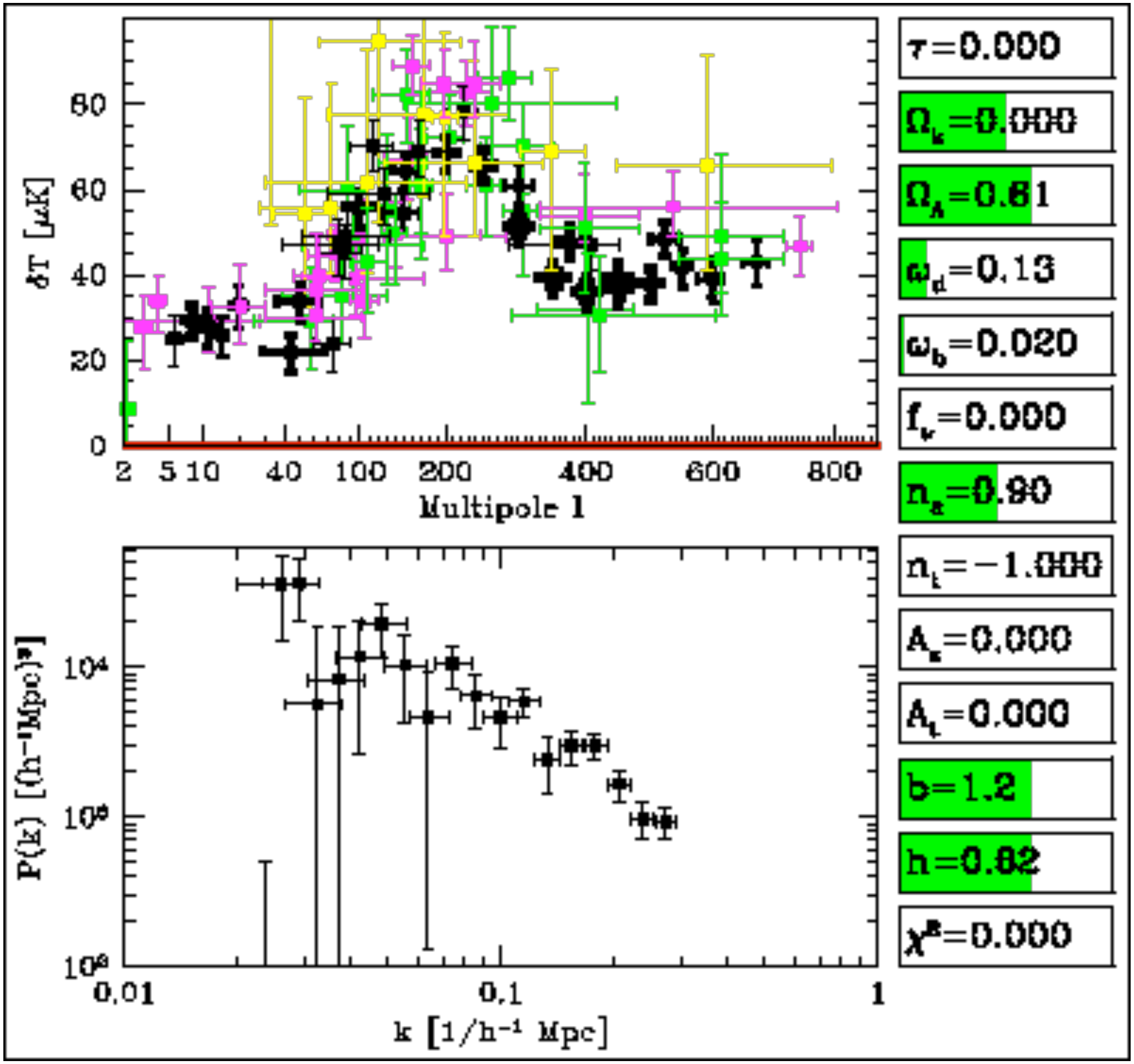
nt=−1.000
As=0.44
At=0.000
b=1.2
h=0.32
χ²=0.000
δT [μK]
Multipole l
P(k) [(h⁻¹Mpc)³]
k [1/h⁻¹ Mpc]



δT [μK]
Multipole l
P(k) [(h⁻¹Mpc)³]
k [1/h⁻¹ Mpc]
τ=0.000
Ωk=0.000
ΩΛ=0.61
ωd=0.13
ωb=0.020
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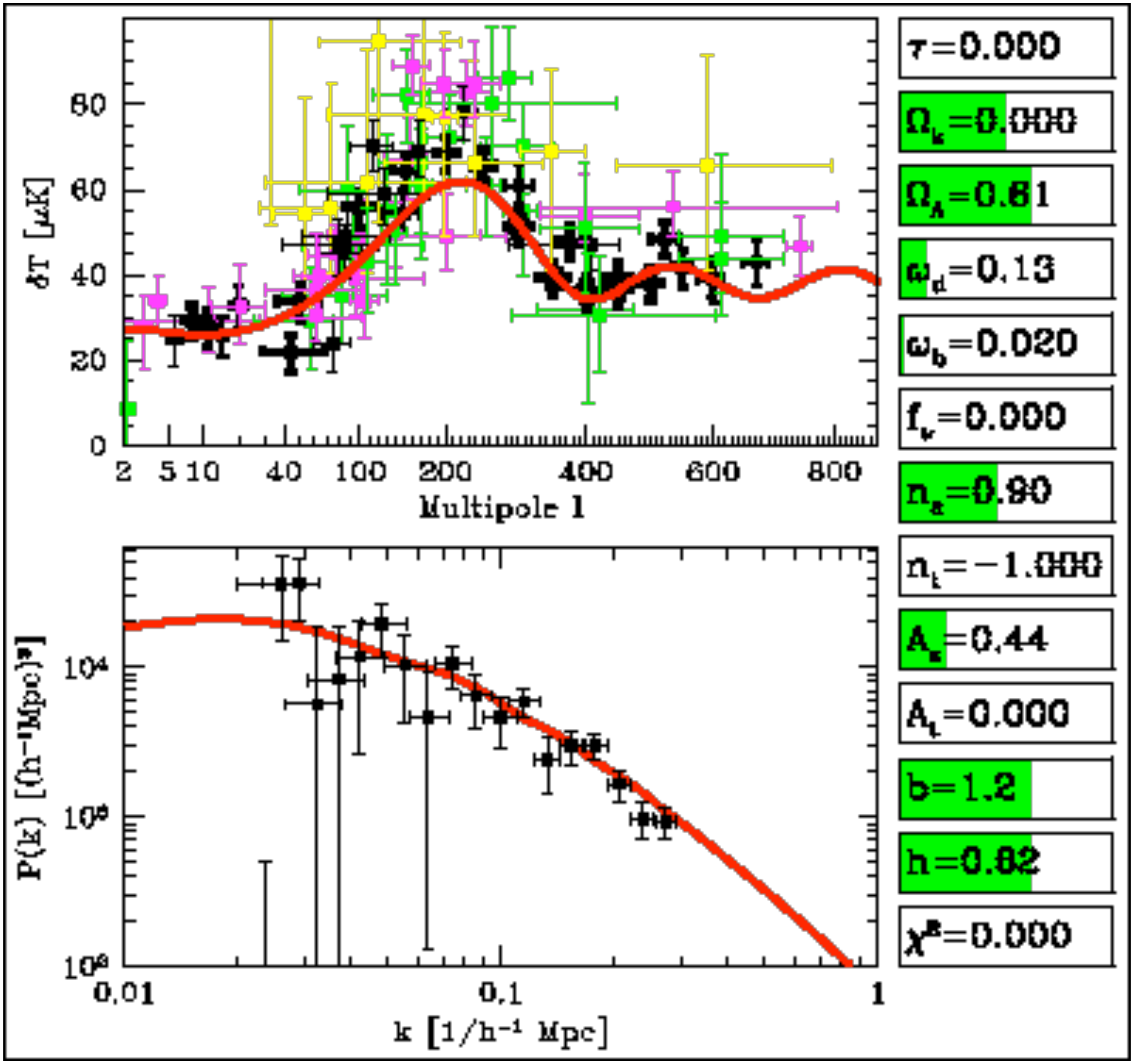
As=0.000
At=0.000
b=1.2
h=0.62
χ²=0.000



δT [μK]
Multipole l
P(k) [(h⁻¹Mpc)³]
k [1/h⁻¹ Mpc]
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ΩΛ=0.81
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ωb=0.020
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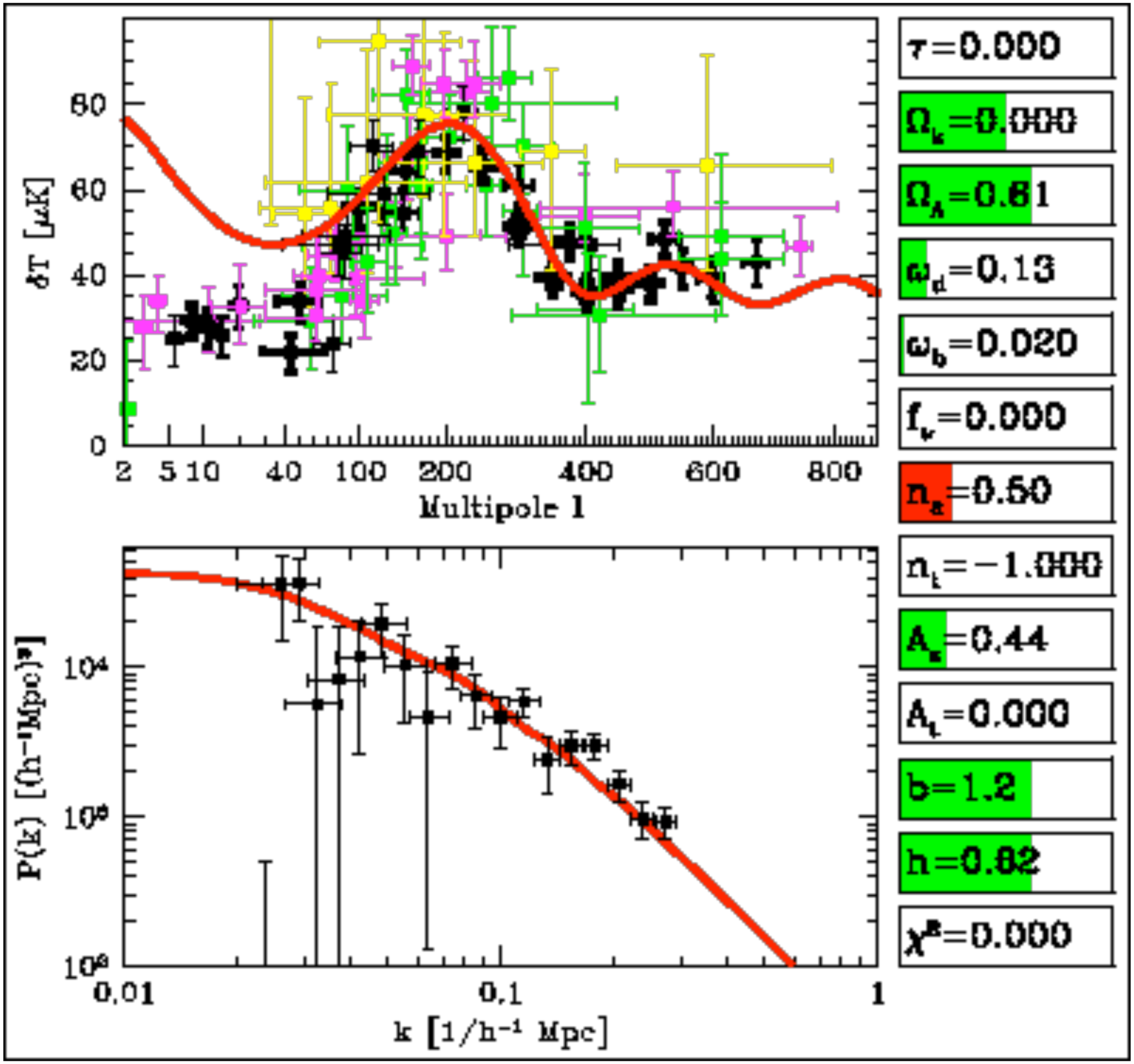
At=0.000
b=1.2
h=0.82
χ²=0.000



δT [μK]
Multipole l
P(k) [(h⁻¹Mpc)³]
k [1/h⁻¹ Mpc]
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Ω_k=0.000
Ω_Λ=0.81
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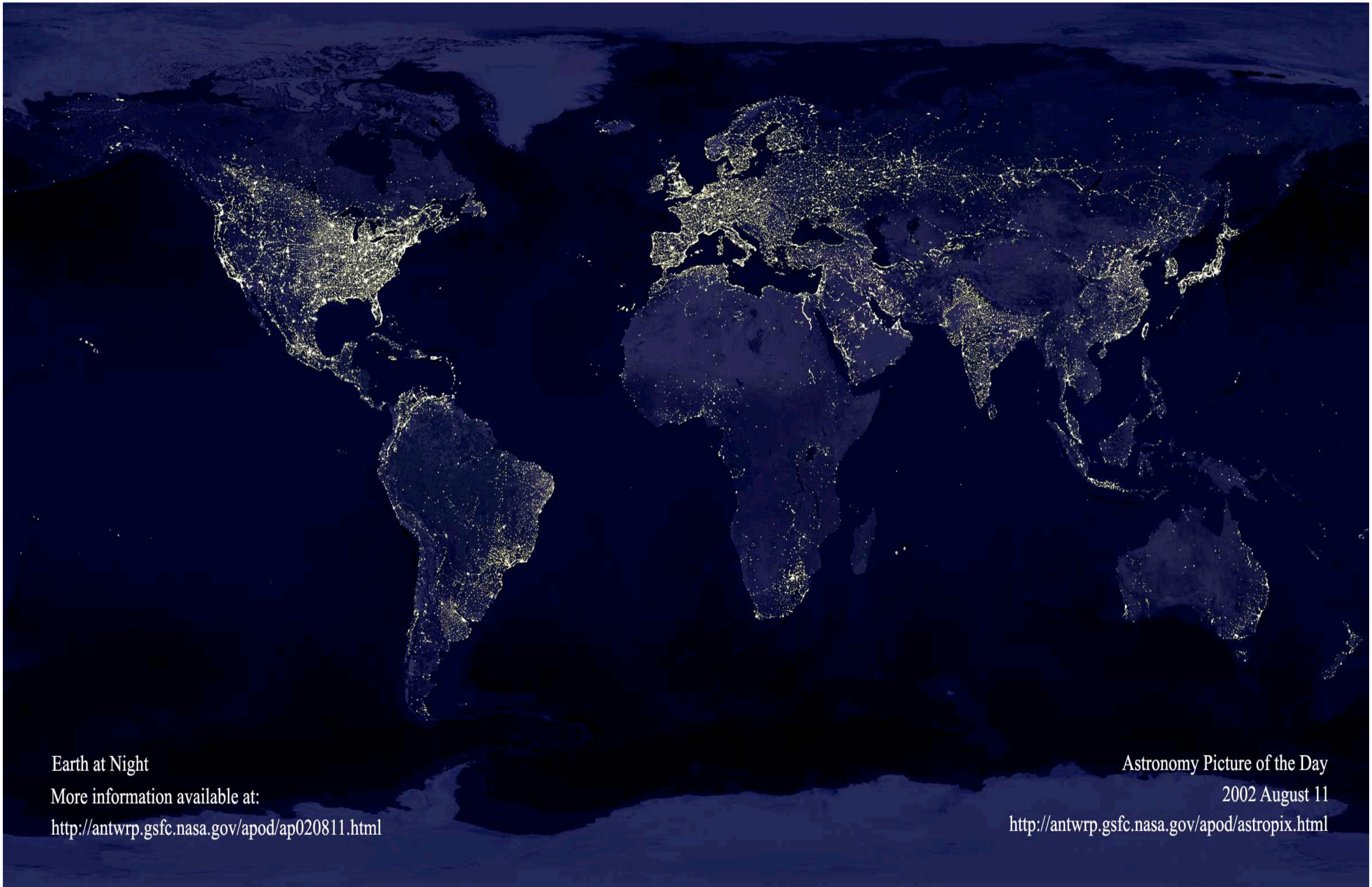
ω_b=0.020
f_ν=0.000
n_s=0.50
n_t=−1.000
A_s=0.44
A_t=0.000
b=1.2
h=0.82
χ²=0.000

Complications…

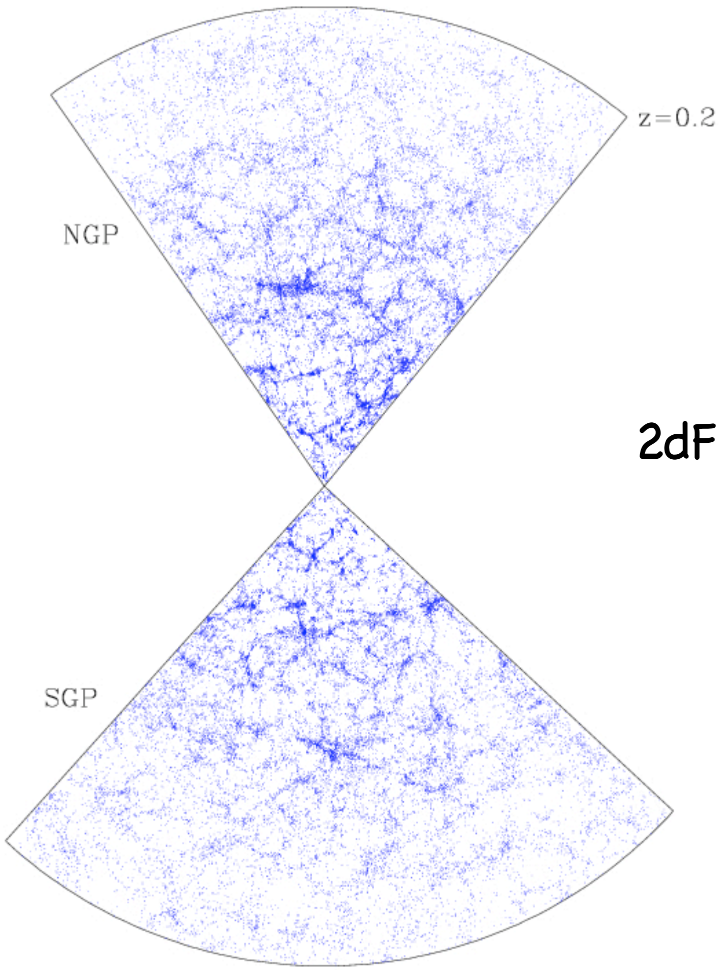
I- Galaxy biasing
(where cosmology meets astrophysics)

Light does not trace mass

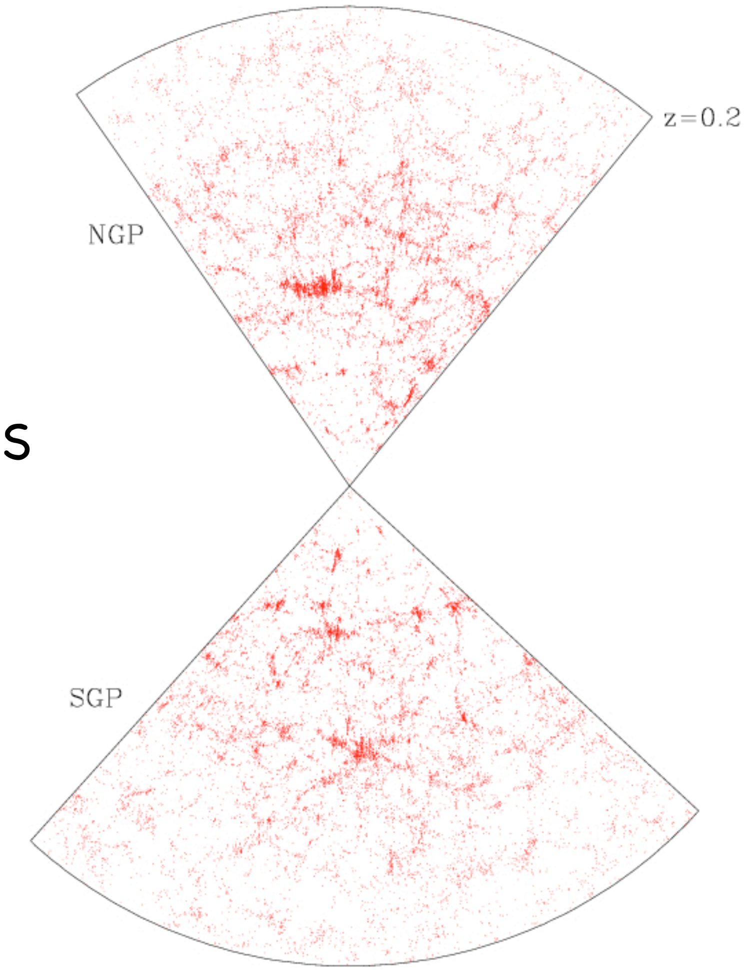
- We observe galaxies and use them to map the cosmic web
- Theory, however, predicts the mass distribution
- So far we have a limited understanding of the galaxy formation process (a complicated (g)astrophysical problem)
- It is clear, anyway, that galaxies form in special regions of the density field with different statistical properties


Earth at Night
More information available at:
http://antwrp.gsfc.nasa.gov/apod/ap020811.html
Astronomy Picture of the Day
2002 August 11
http://antwrp.gsfc.nasa.gov/apod/astropix.html

Galaxy biasing exists

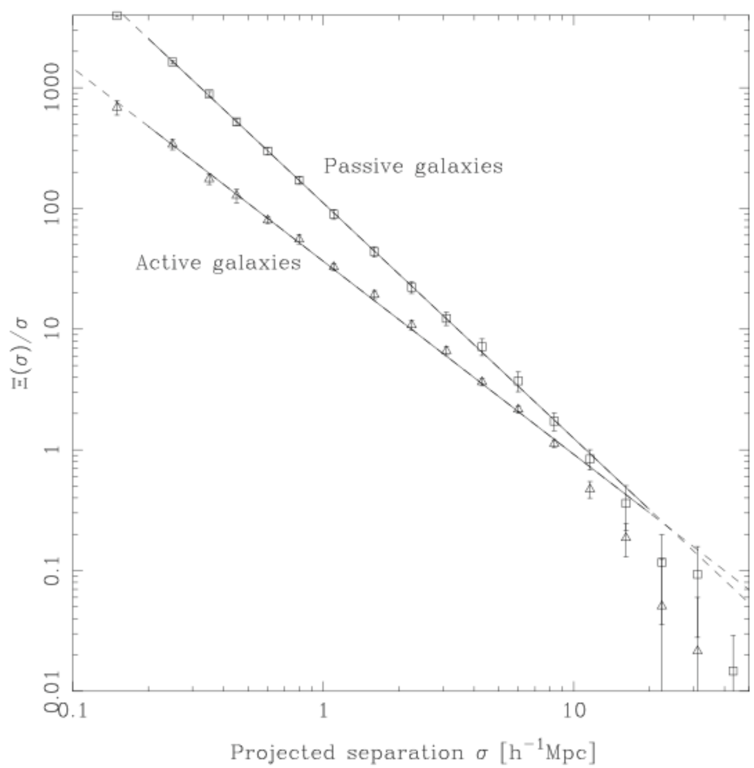


2dF GRS

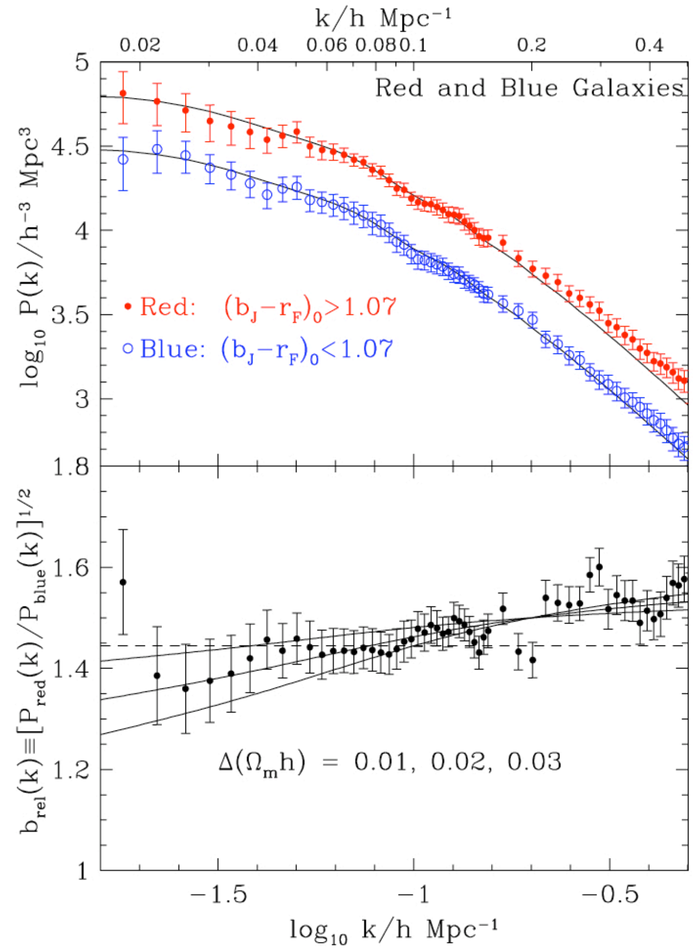


Galaxy biasing exists

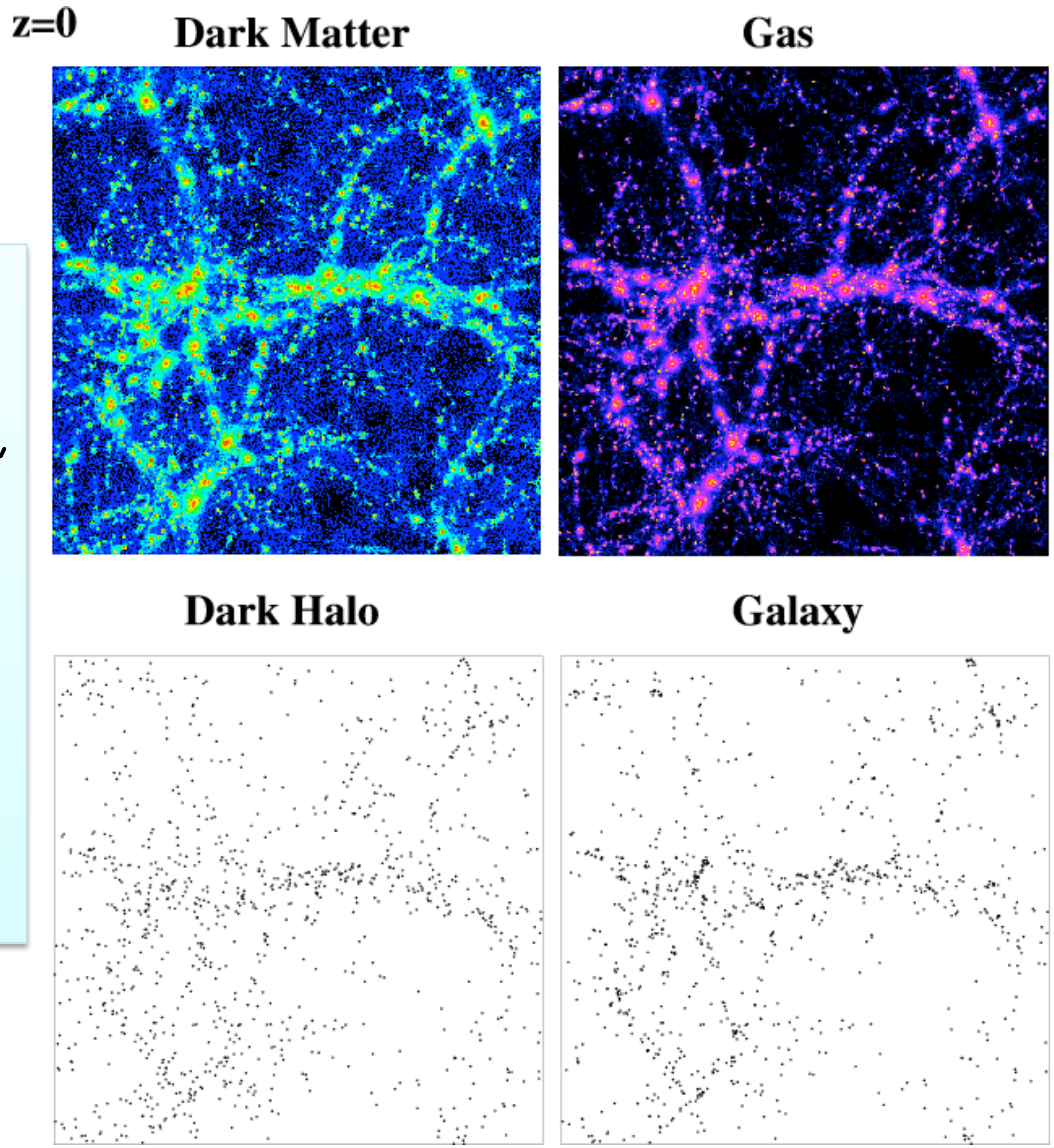
Madgwick et al. 2003



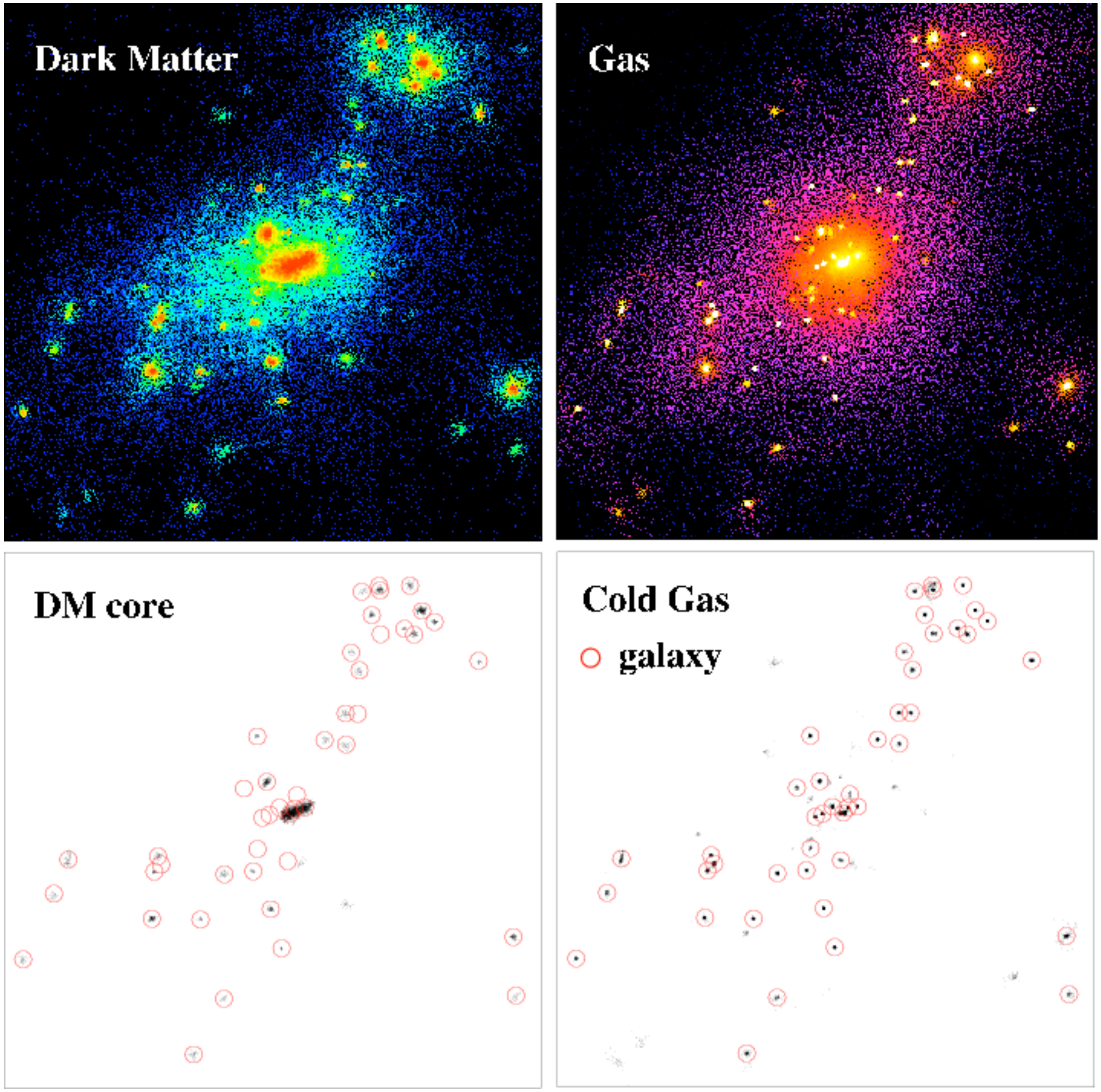
Cole et al. 2005



To better understand how to model galaxy biasing, we can compare the distribution of different tracers of the large-scale structure in a numerical simulation



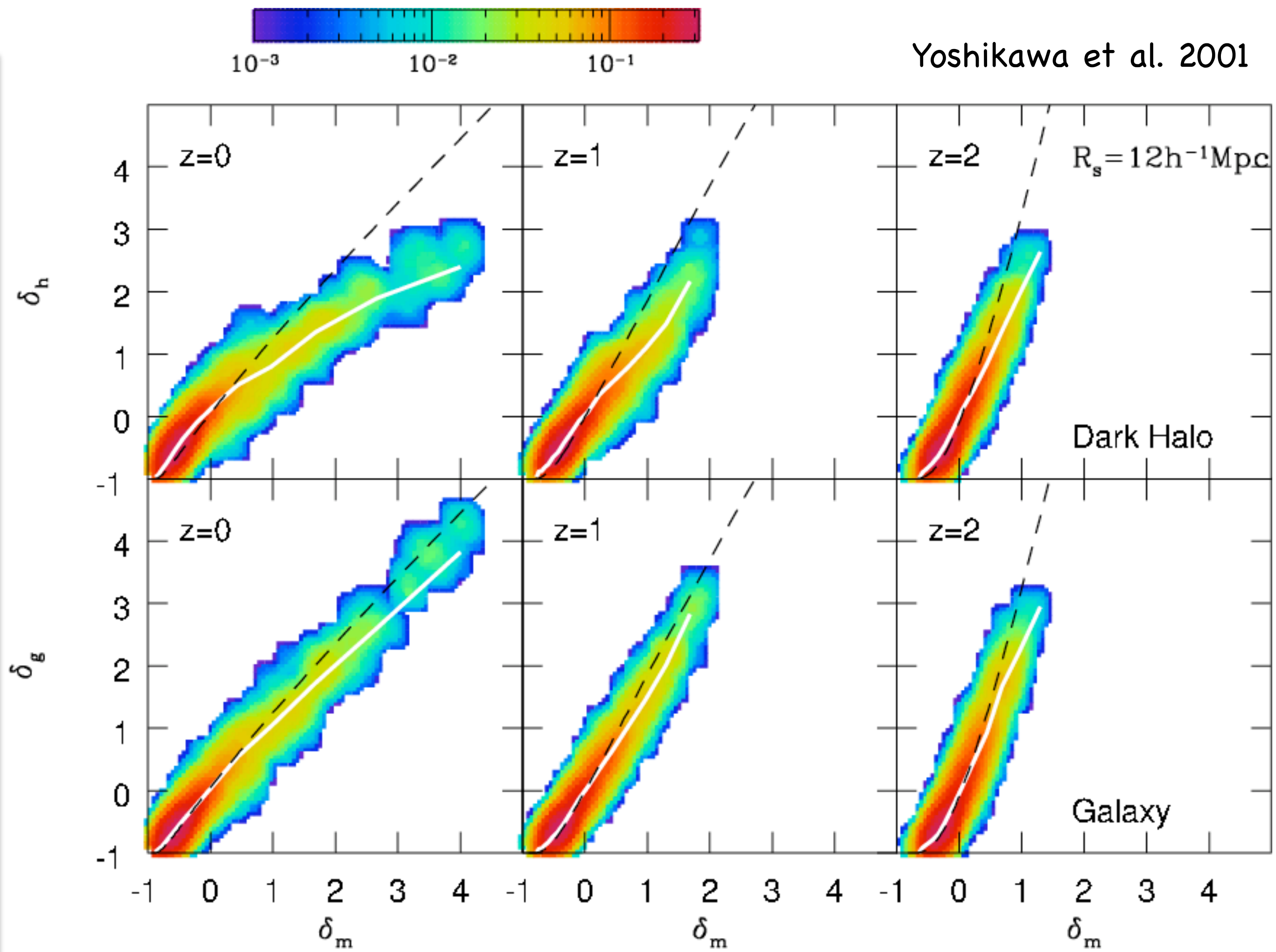
Zooming in a galaxy cluster



A local biasing scheme?

Smooth the density distributions of different tracers on the scale R_s and plot them against the mass density (also smoothed) at the same spatial location. Apart from some scatter there appears to be a deterministic relation.

Yoshikawa et al. 2001



A local biasing scheme

- Therefore, we can write that

Scatter, noise

$$\delta_g(x) = f[\delta_m(x)] + \varepsilon(x)$$

- And, for a large smoothing scale, for which $\delta_m \ll 1$ we can Taylor expand the deterministic part and write (neglecting the scatter)

$$\delta_g(x) \approx b_0 + b_1\delta_m(x) + \frac{b_2}{2}[\delta_m(x)]^2 + \ldots$$

- This implies that

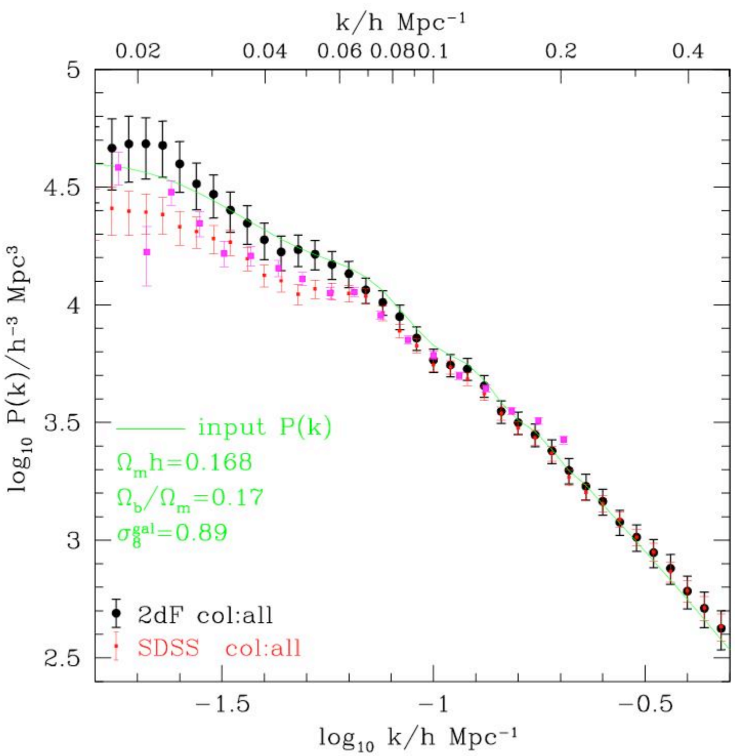
$$P_g(k) = b_1^2 P_m(k) + g[b_1, b_2, P_m(k)]$$

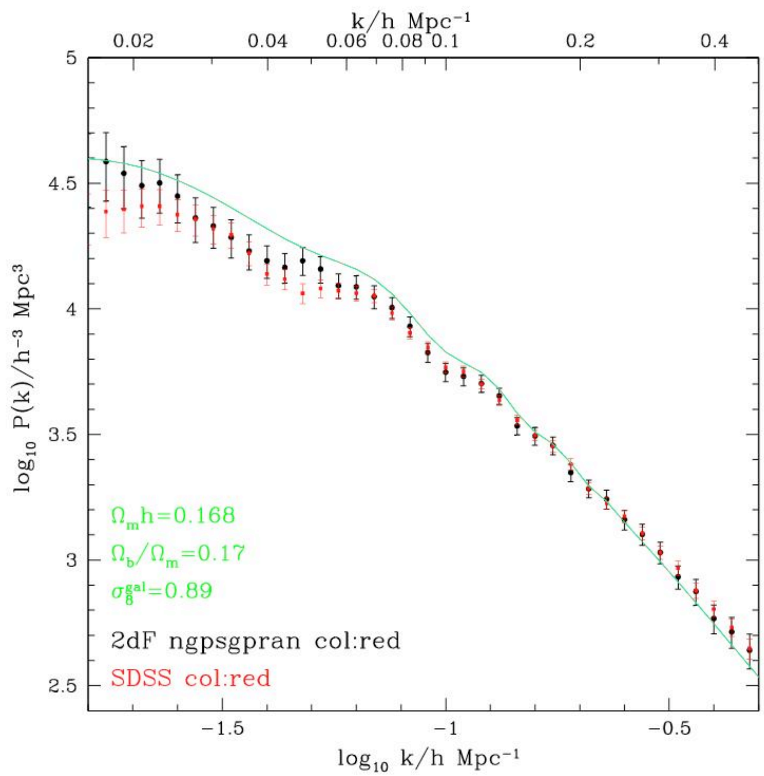
Linear biasing term: changes the amplitude

Non-linear biasing term: changes the shape

Power spectrum and galaxy selection 2dF GRS vs SDSS

Cole et al. 2005

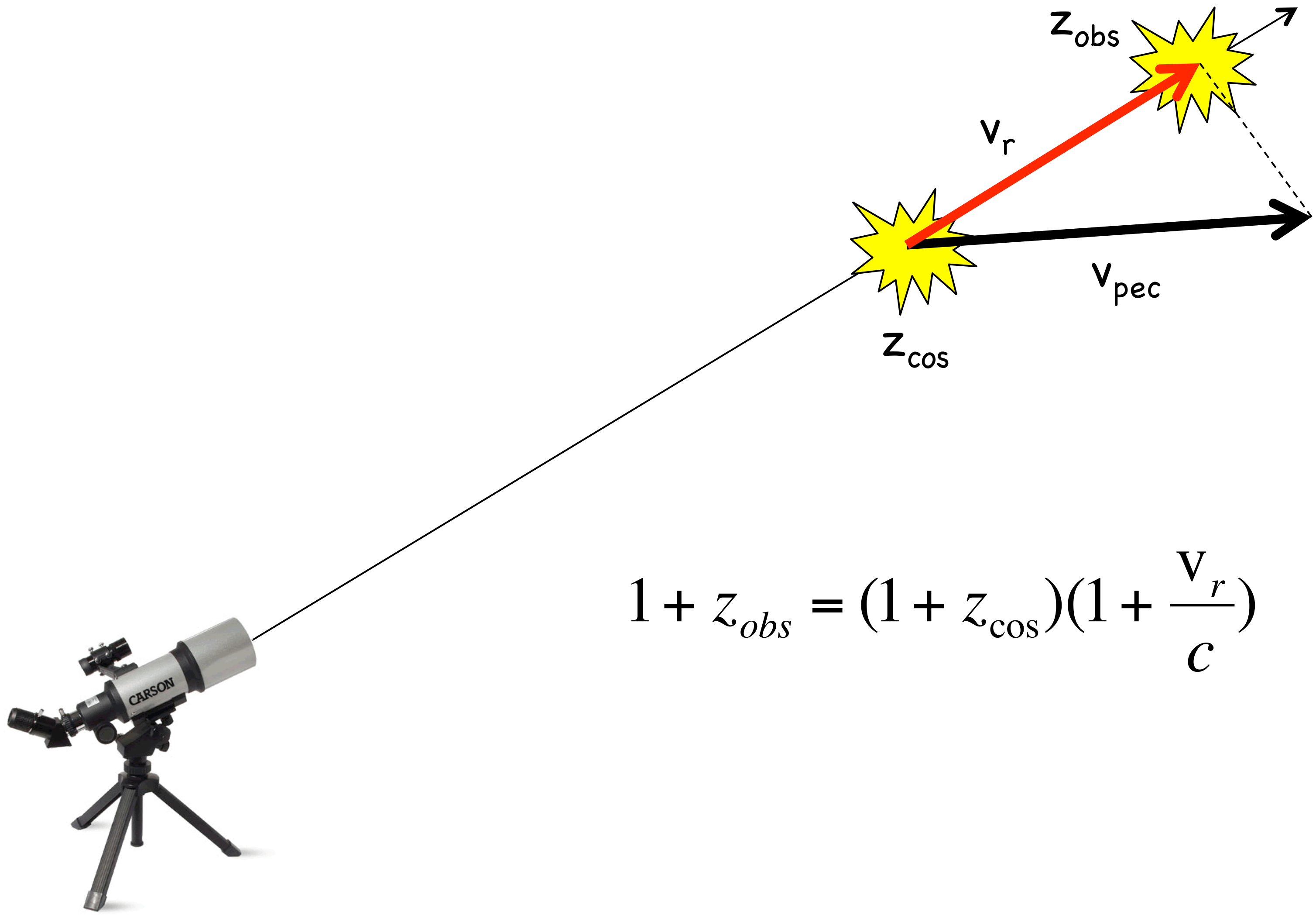




Complications…

II- Redshift-space distortions

Redshift-space distortions



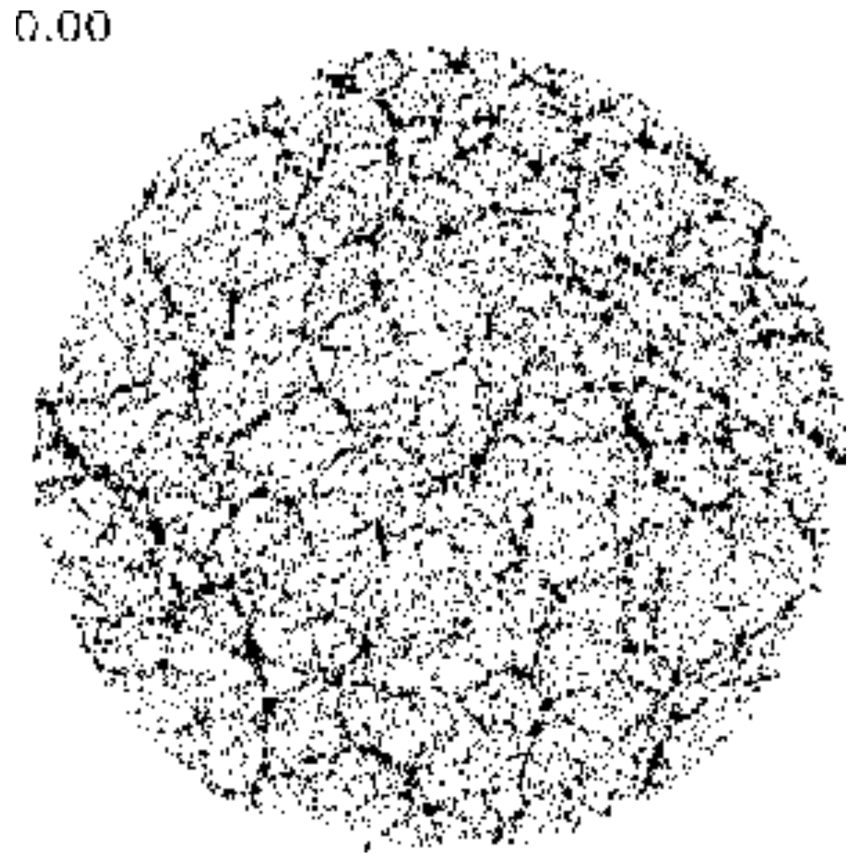
$$1 + z_{obs} = (1 + z_{\cos})(1 + \frac{v_r}{c})$$

Redshift-space distortions

- Density fluctuations generate velocities on top of the global cosmic expansion

- The observed redshift of a galaxy includes a radial Doppler component: $1+z_{obs}= (1+z_{cos})\ (1+v_r/c)$

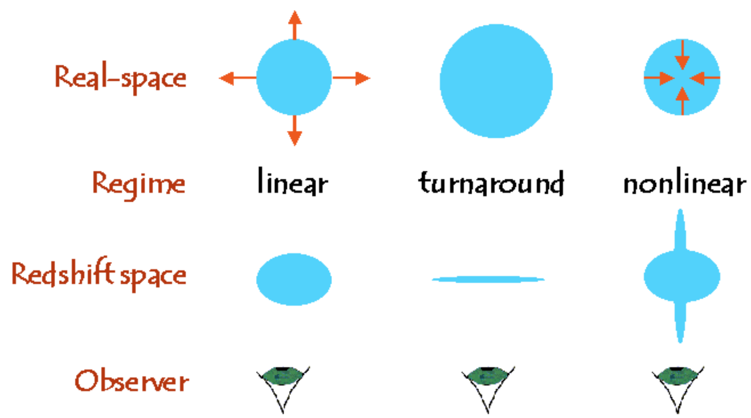
- Since we use the redshift to infer the distance to a galaxy, our 3D maps of the universe are "distorted".



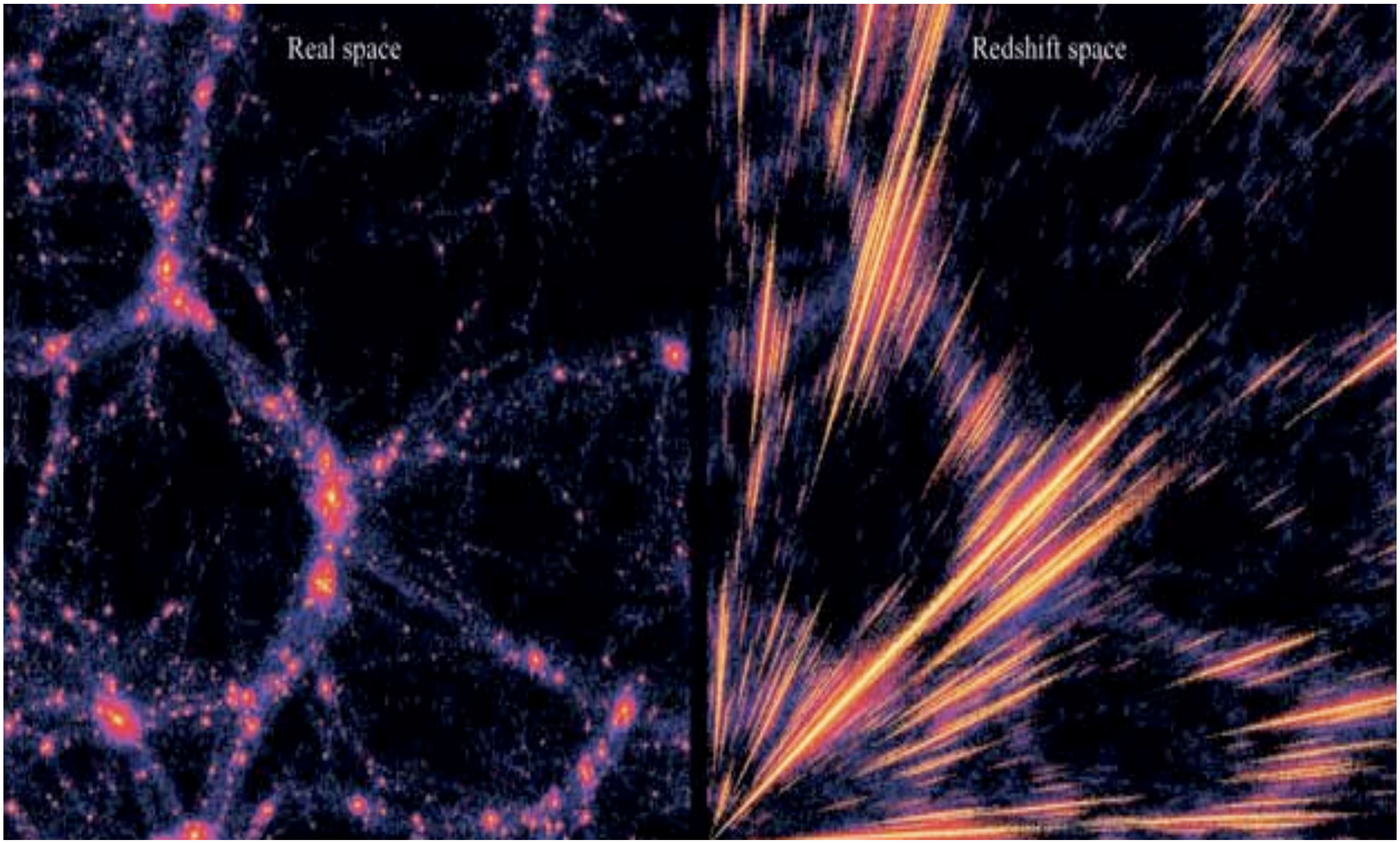
Redshift distortions

- Fingers of God: Radial stretching pointing towards the observer. They come about because of random velocities in clusters of galaxies

- Large overdensities lead to a coherent infall motion: walls appear denser and thicker, voids bigger and emptier



A closer look



Consequences of RS-distortions

- Bad news: we will never be able to measure the actual galaxy distribution

- Good news: the size of the distortions depends on cosmology. We can use them to learn something about the universe. Recall from cosmology class:

$$\nabla \cdot \mathrm{v} = -\frac{\partial \delta_m}{\partial t} \text{ (linearized continuity equation)}$$

$$\delta_{g,s}(\vec{k}) = \delta_{g,r}(\vec{k}) \left(1 + \beta\mu^2\right), \quad \beta \approx \frac{\Omega_m^{0.55}}{b_1}, \quad \mu = \cos(\theta_{\vec{r}\vec{k}})$$

The power spectrum in redshift space

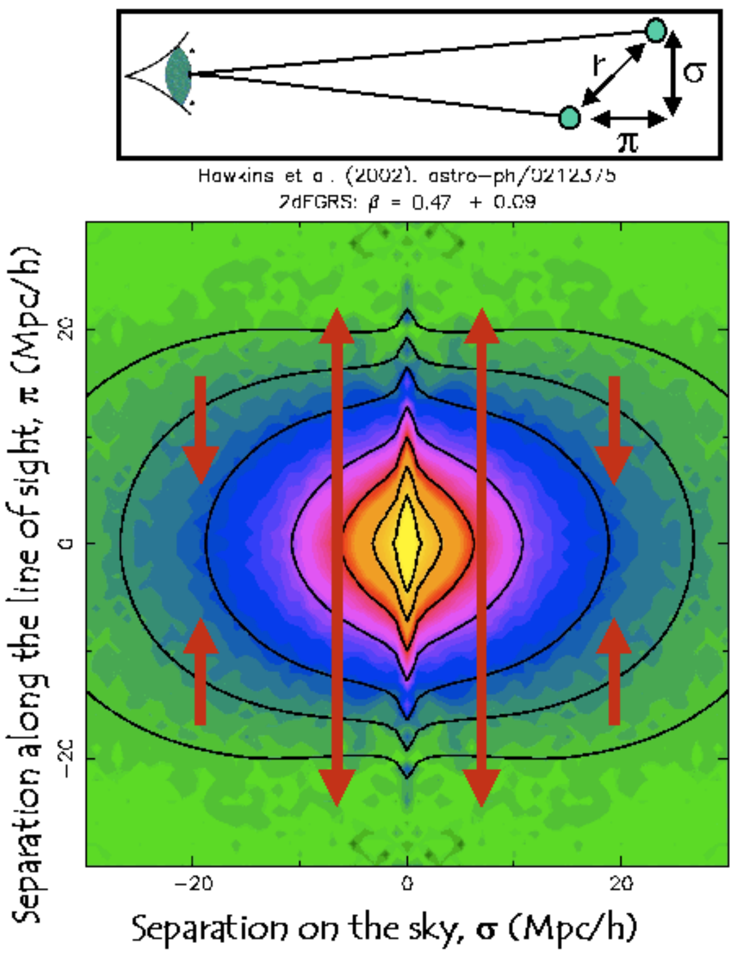
Boost in the average power + anisotropic terms

$$\frac{P_s(k)}{P(k)} = \left(1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2\right) + \left(\frac{4}{3}\beta + \frac{4}{7}\beta^2\right) L_2(\mu) + \frac{8}{35}\beta^2 L_4(\mu)$$

where $L_i(x)$ denotes the Legendre polynomial of order i

The ratio of the quadrupole to monopole amplitudes is a monotonic function of β that rises from 0 at $\beta=0$ to just over unity at $\beta=1$. Redshift distortions can then be used to measure β and, if one already knows b_1, provide a measure of Ω_m

Anisotropic correlation function



- Redshift distortions also generate anisotropies in the 2-point correlation function
- The finger-of-god effect can be used to determine the velocity dispersion (and thus the typical mass) of the galaxy groups
- The squashing effect on large scales is equivalent to the quadrupole to monopole rato in the power spectrum and can be used to further constrain the cosmological model

Questions?

Complications…

III- Shot noise

Shot noise

- Galaxies are discrete objects
- For mathematical convenience, we describe their distribution with a continuous random field that it is sampled at random positions (note that there are 2 levels of randomness here)
- The effect of the sampling it is called shot noise and we need a model for it (there are infinite ways to do it). The most used is Poisson sampling (but never forget that it is just an approximation):

$$P(N\,|\,\delta) = \mathrm{Poisson}[(1+\delta_{gal})\bar{n}_{gal}V]$$

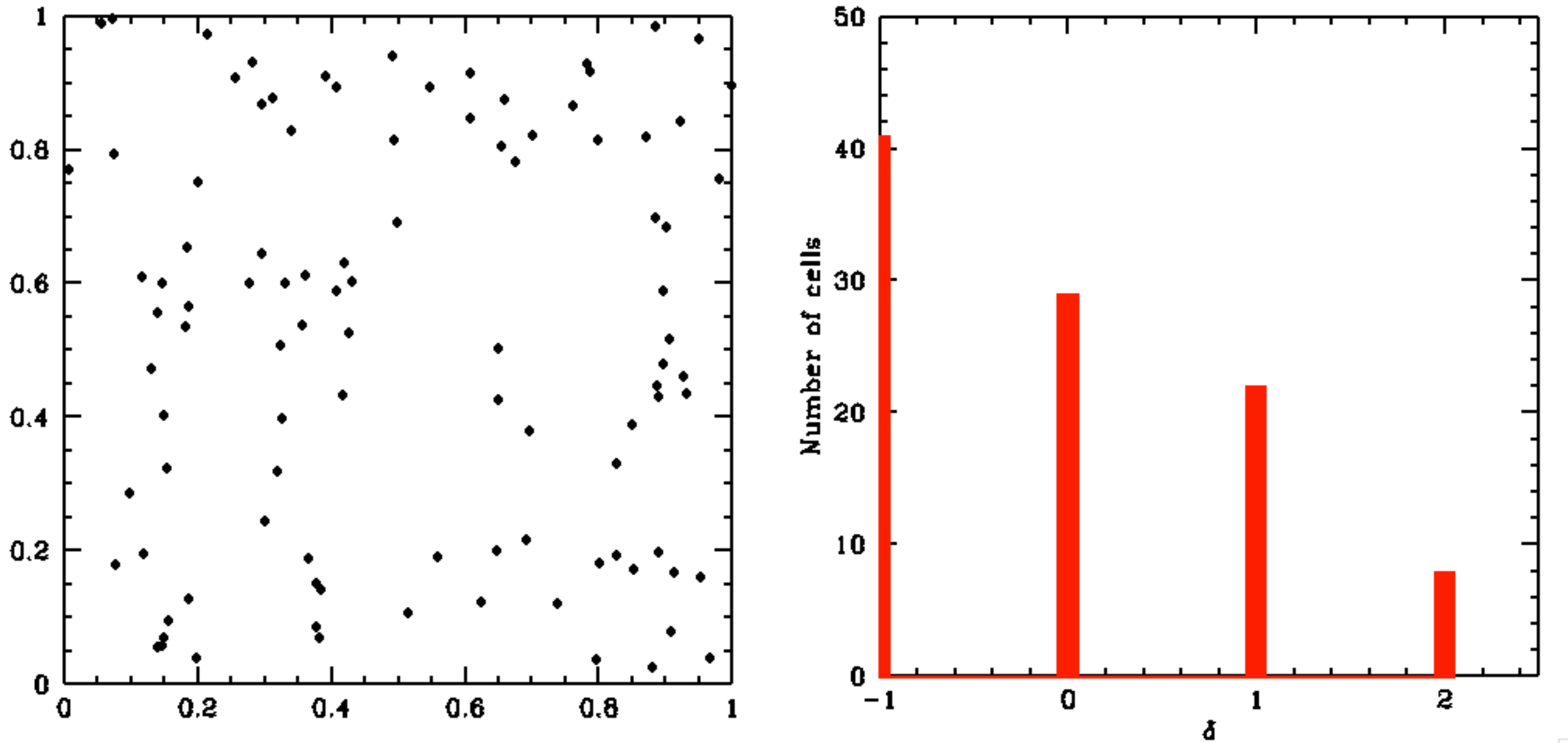
where P(N| δ) gives the probability of finding N galaxies in a volume V with underlying "continous" overdensity δ

Shot noise

- Shot noise also refers to the effect of self pairs (i.e. pairs made by a single objects) in N-point statistics

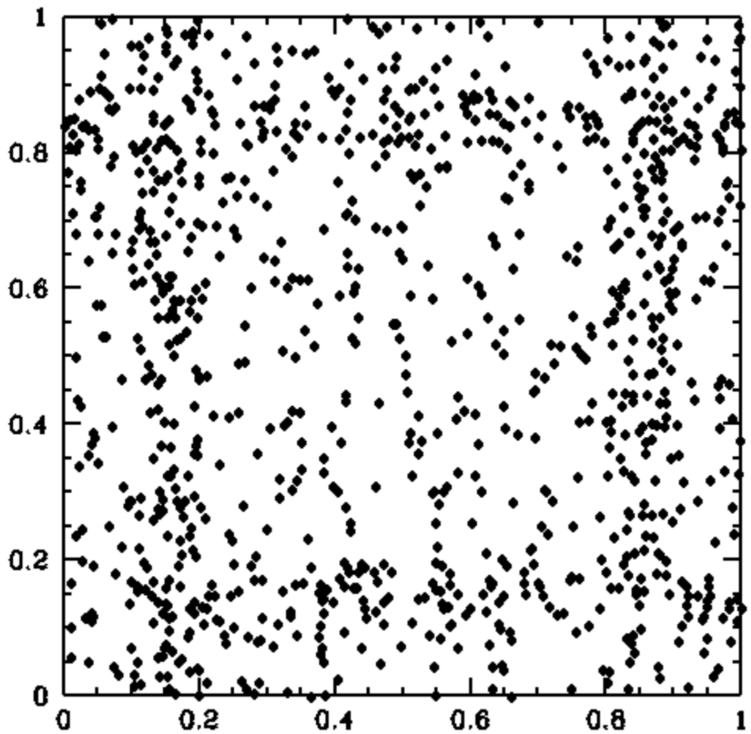
Shot noise: an example

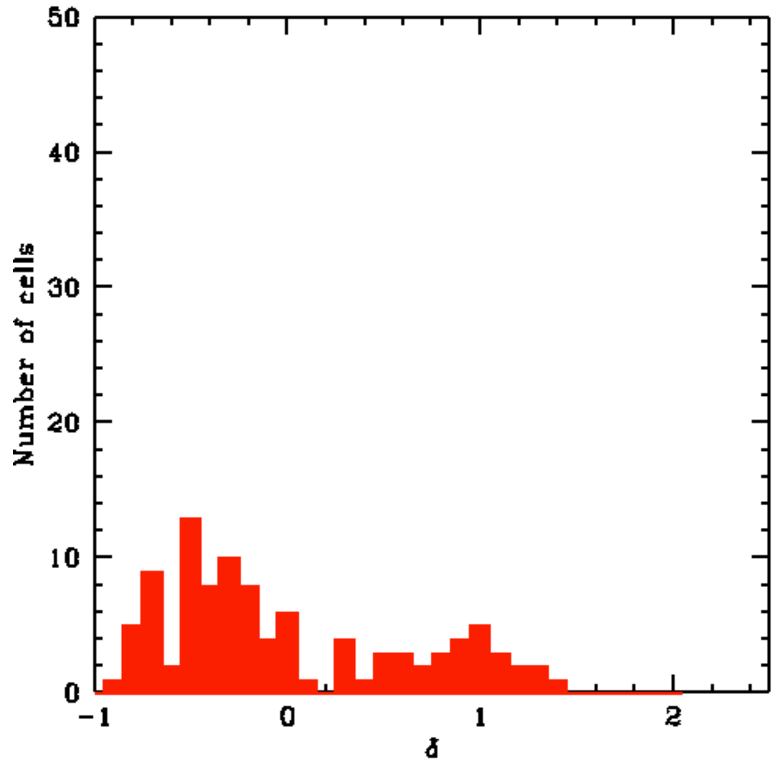
100 tracers



Shot noise: an example

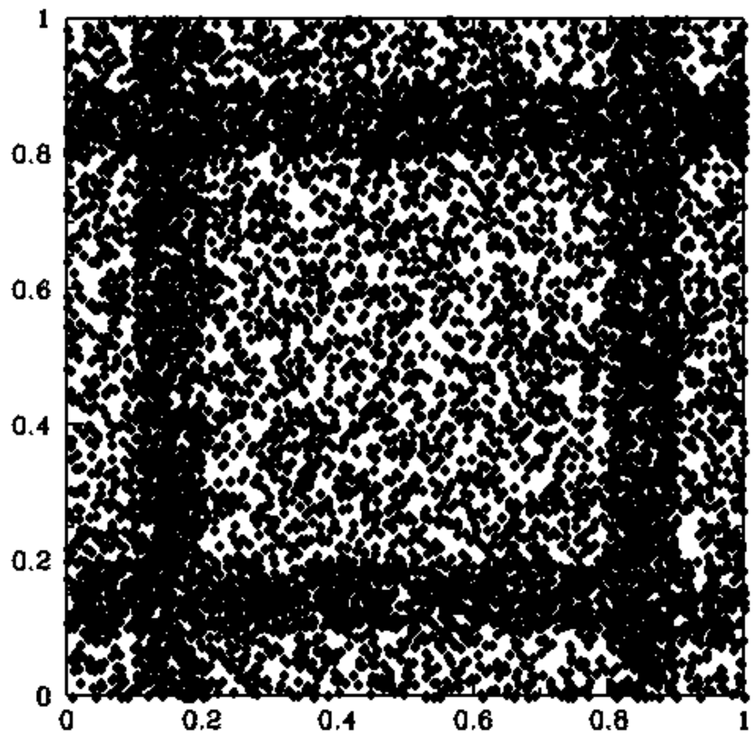
1000 tracers

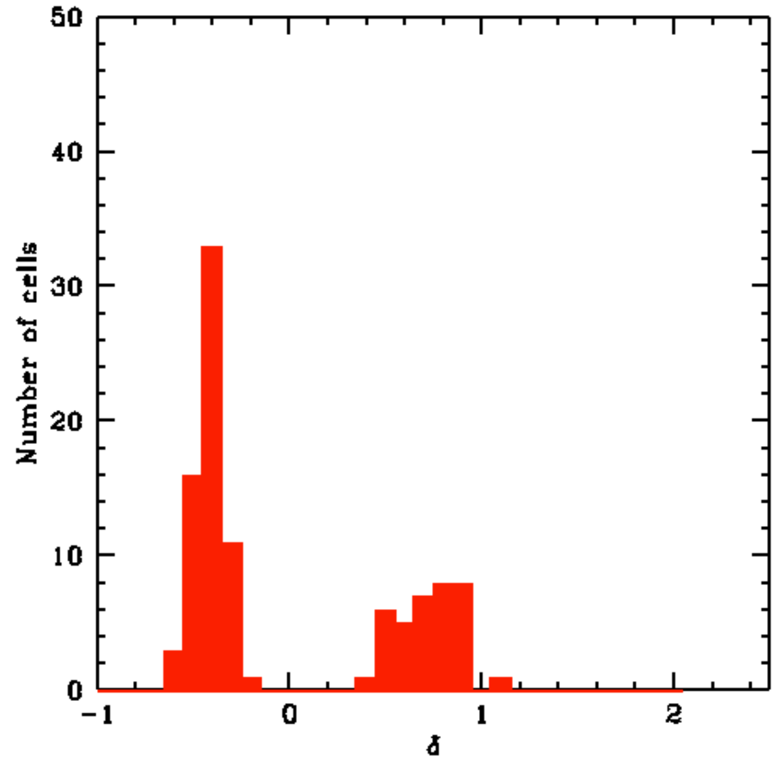




Shot noise: an example

10000 tracers





Shot noise and power spectra

- Poissonian shot noise affects power spectra in two ways

- First it adds a (white) systematic component

$$P_{obs}(k) = P(k) + \frac{1}{\bar{n}_{gal}}$$

- Second, it increases statistical uncertainties

$$\frac{\sigma_{P(k)}}{P(k)} = \left(\frac{2}{N}\right)^{1/2}\left(1 + \frac{1}{\bar{n}_{gal}P(k)}\right)$$

N = number of modes in a k-bin (scales as the volume of the survey)

Sample variance (for a Gaussian field)

Shot noise

Question

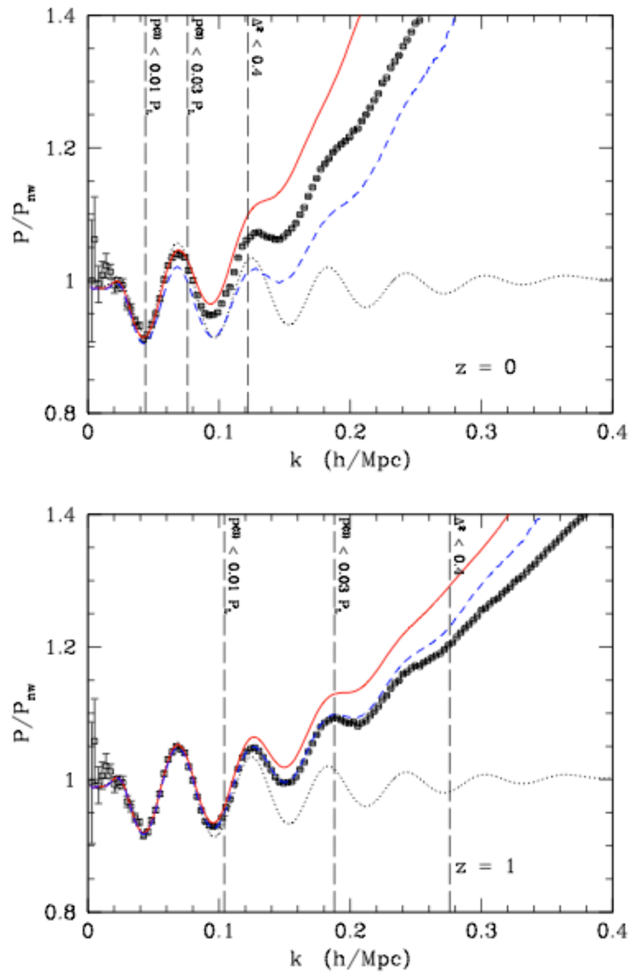
- What is the effect of shot noise on the 2-point correlation function?

Complications…

IV- Non-linear evolution

Non-linear evolution of the mass power spectrum

Carlson et al. 2009



- The non-linear growth of density perturbations changes the shape of their power spectrum from the linear one

- Current models are not very precise in recovering this behaviour for k>>0.1 h/Mpc

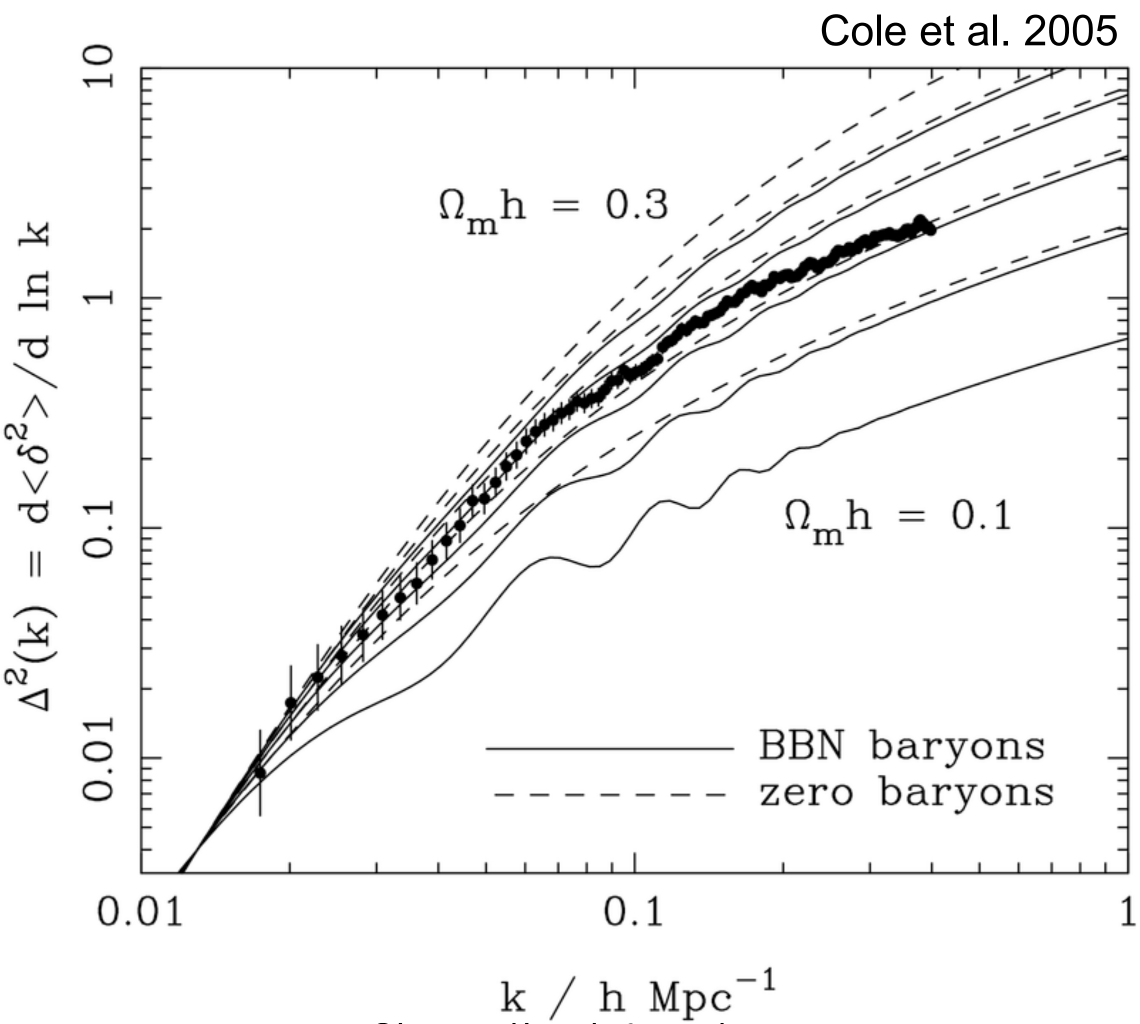
Outstanding question

- Do uncertainties in modelling non-linearity, redshift distortions and galaxy bias compromise constraints on cosmological parameters coming from measurements of the galaxy power spectrum?

- Answer: they do not as long as we just use data on very large scales where linear models (for bias and for the evolution of perturbations) are accurate enough.

- This, however, makes errorbars of cosmological parameters big (with respect to the potential of the data) and a lot of efforts are currently made to improve the modelling of the non-linear effects

Cosmology from galaxy clustering



What did we learn?

- On separations larger than a few Mpc, models show that the ratio between the matter power spectrum and the galaxy power spectrum is nearly constant

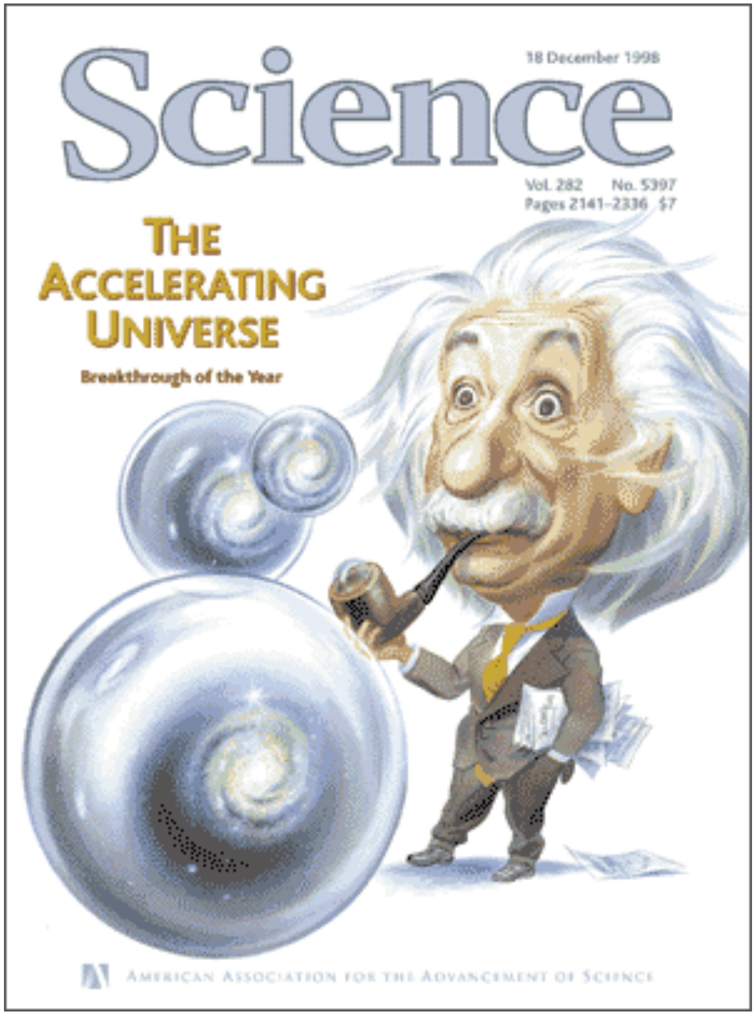
- This implies that we can use the shape of the galaxy power spectrum to determine the cosmology

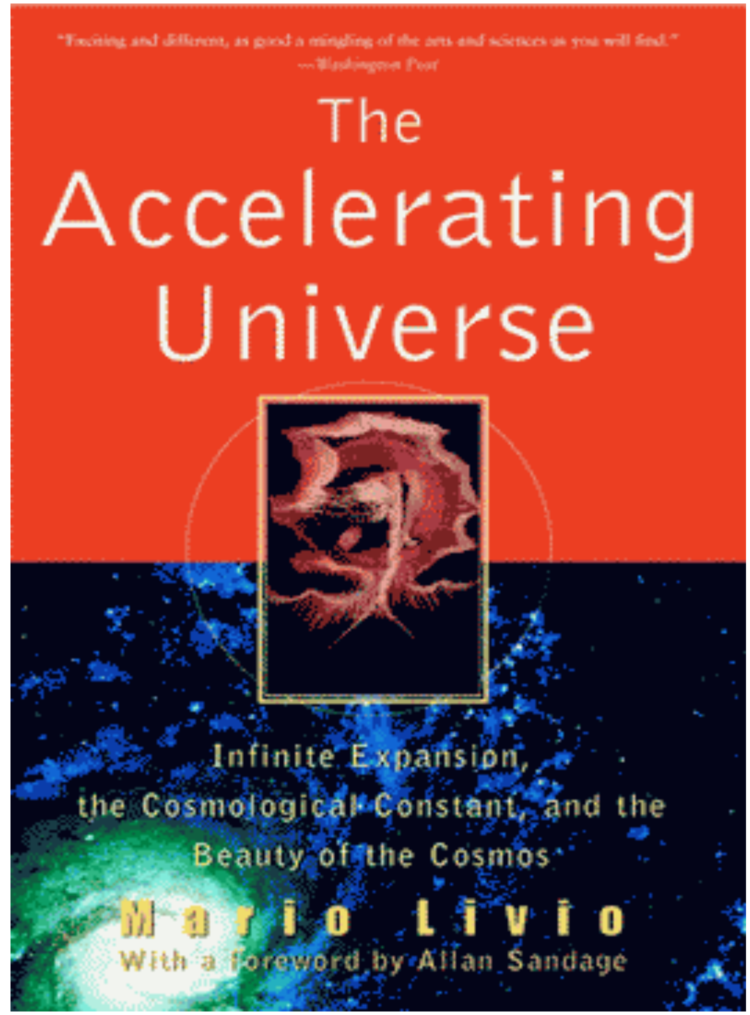
- Galaxy clustering gives $\Omega_m h \approx 0.2$, which for an Hubble constant $h=0.7$ gives $\Omega_m \approx 0.25-0.3$

- Combining this with the results of the CMB ($\Omega_{tot} \approx 1$), it suggests that 75% of the energy in the universe is in an unknown form, the so-called dark energy

Constraining dark energy with galaxy redshift surveys

A surprise

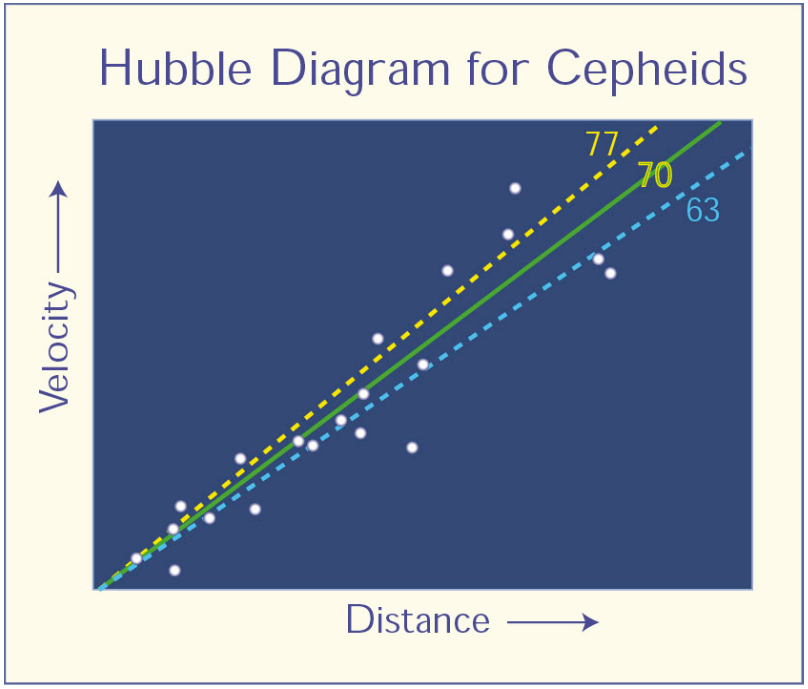

Science
18 December 1998
Vol. 282 No. 5397
Pages 2141–2336 $7
The Accelerating Universe
Breakthrough of the Year
American Association for the Advancement of Science


The Accelerating Universe
Infinite Expansion, the Cosmological Constant, and the Beauty of the Cosmos
Mario Livio
With a foreword by Allan Sandage

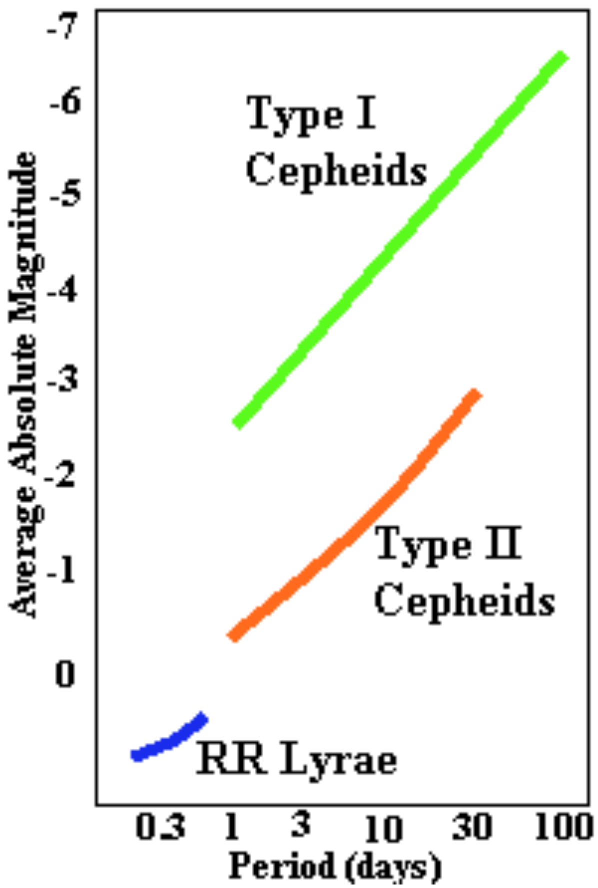
Luminosity distance and standard candles

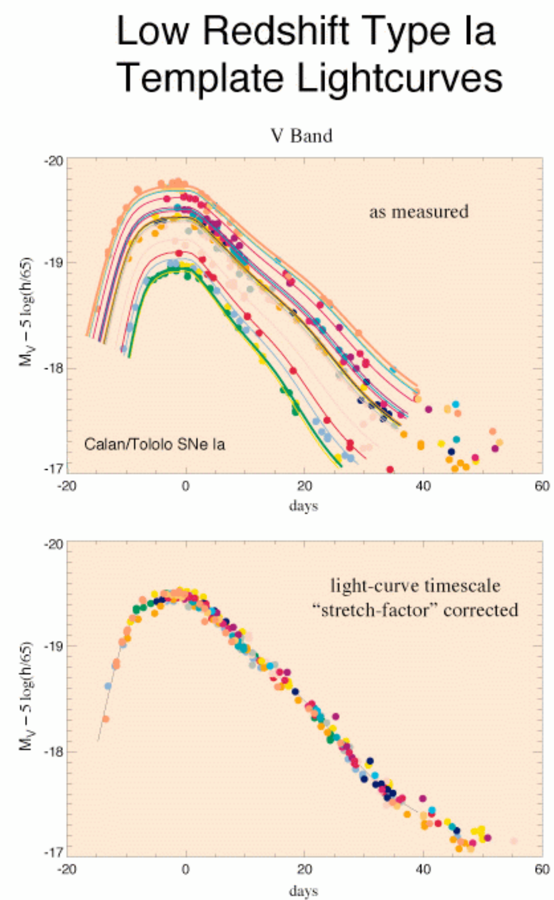
The Hubble diagram

- Taylor expansion of the lumin. distance-redshift relation: $H_0 d_L/c= z +(1-q_0)z^2/2+ \ldots$
- This is the observational version of the Hubble's law
- It is very difficult to measure distances on cosmological scales
- Need for standard or standardizable candles
- The best we have today are Cepheid stars (PL or PLC relation) and Supernovae Ia (peak brightness - decay time relation)

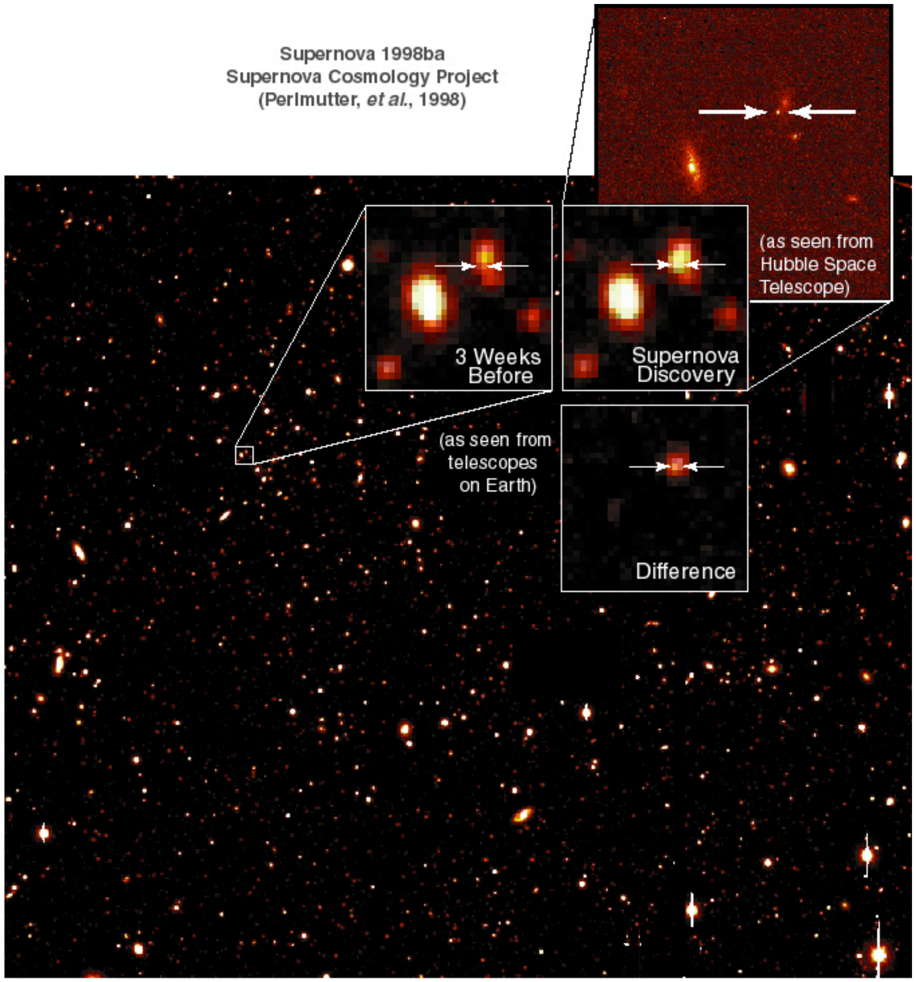


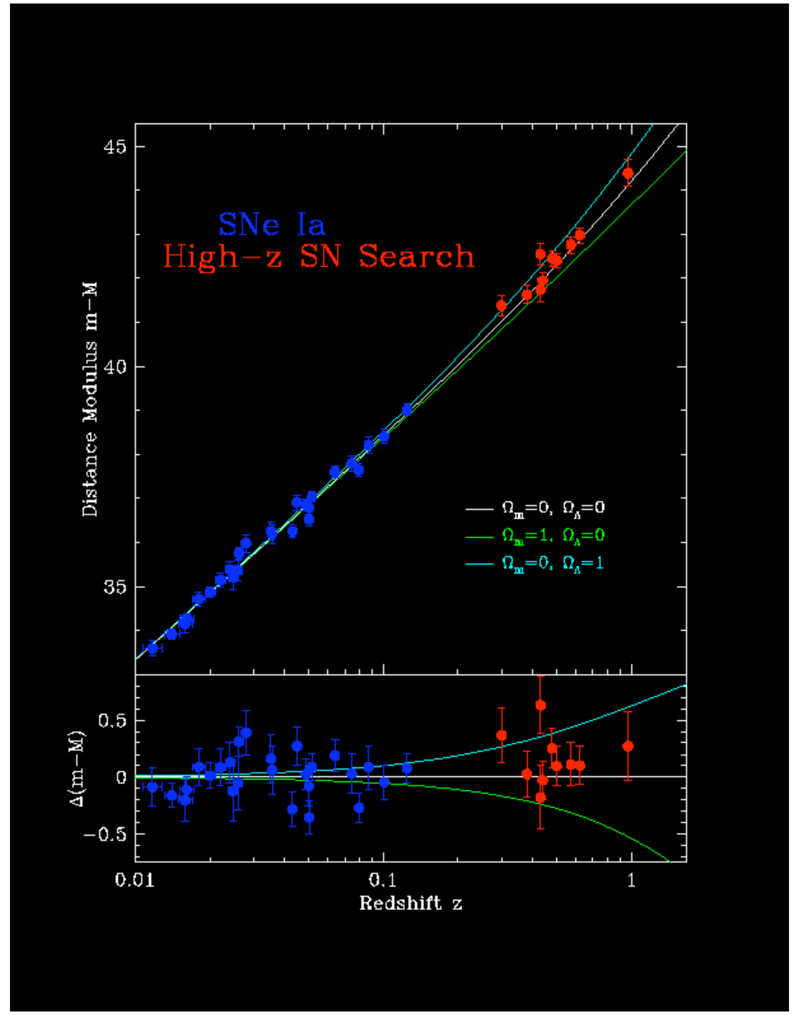
Standardizable candles





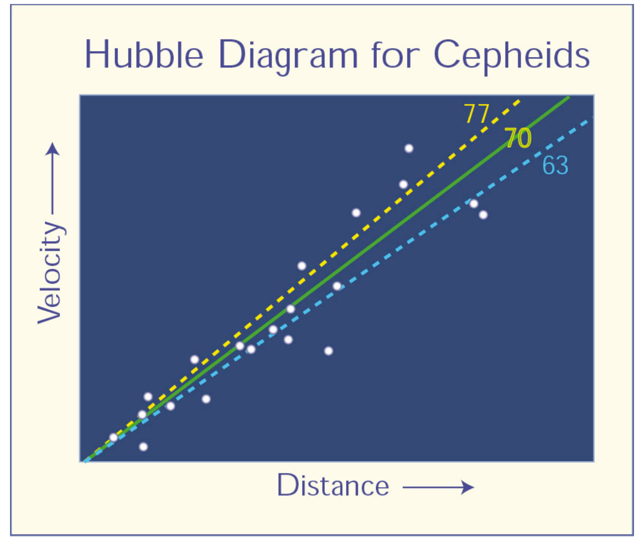
Hubble diagram from SNae Ia

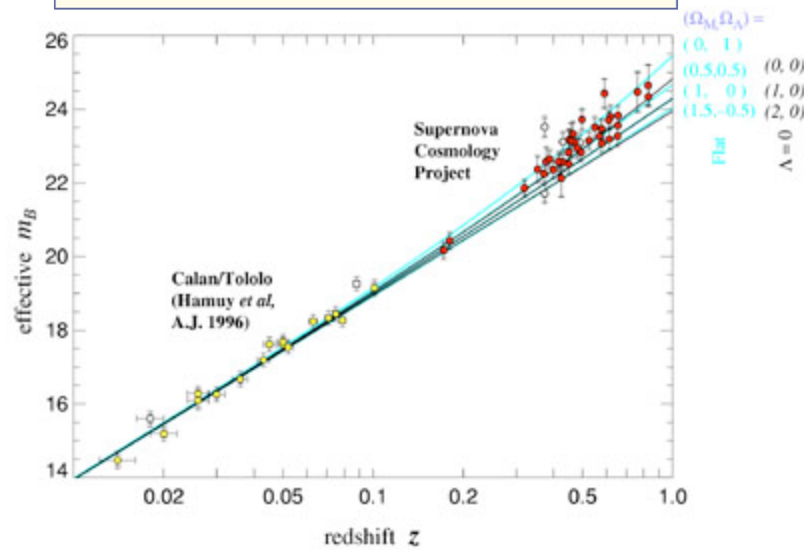




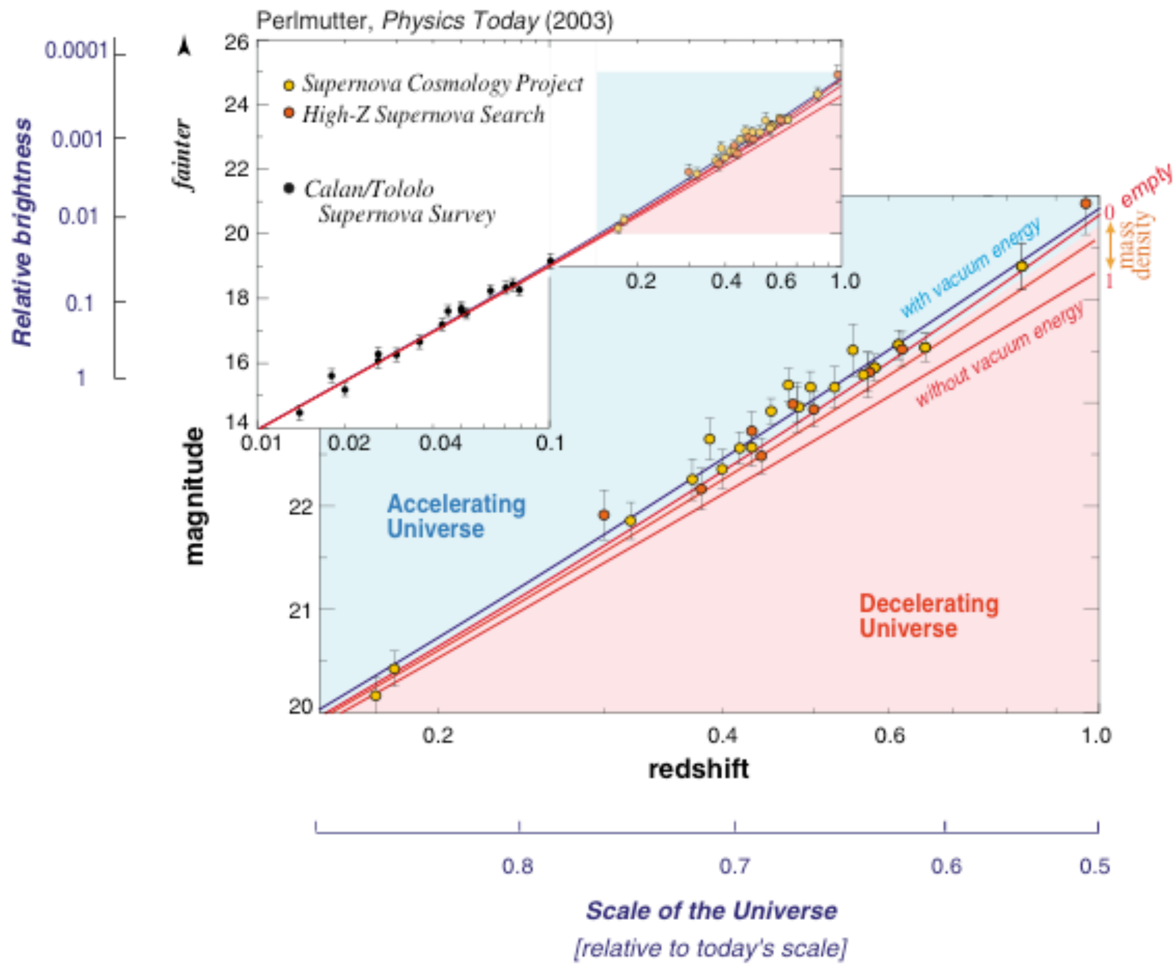
Current estimates

- Improvement: Hipparcos accurate determination of the parallax of local Cepheids
- HST key project (based on Cepheids)
  H_0 = 72 ± 8 km/s/Mpc
  (Freedman et al. 2001)
- Hubble diagram with SNa Ia
  H_0 = 73 ± 7 km/s/Mpc
  (Riess et al. 2005)
- Other estimates from different datasets lie in the same ballpark
- This sets the size and age of the observable universe





Accelerated expansion?

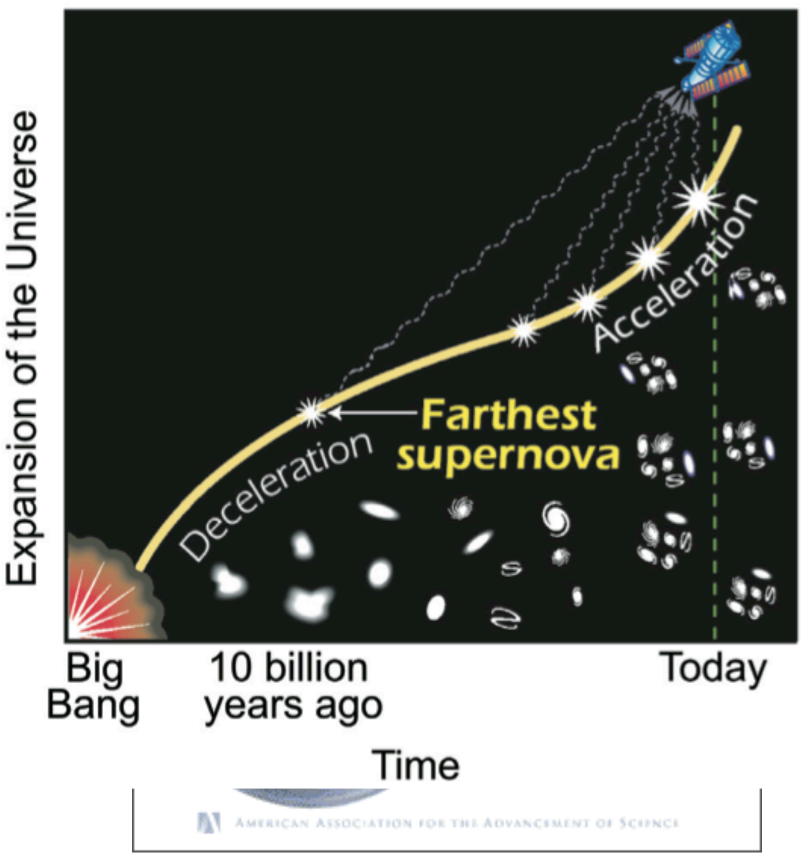


- In 1998, two independent teams found that SNae Ia at z ≈ 0.5 appear about 25% dimmer than they would in a decelerated universe

- This suggests an accelerated Hubble flow: acceleration increases the distance the light must travel to reach us

- Improved data collected in the last few years have confirmed the original results

Dark energy, a primer



- Acceleration of cosmic expansion discovered in 1998 from observation of the distance-redshift relation of supernovae Ia

- Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

then implies $p < -\rho c^2 / 3$ (i.e. a strongly negative pressure or tension)

- The (hypothetical) dominant negative pressure component has been dubbed "dark energy" (name coined by M. Turner)

What could it be?

- The cosmological constant, Λ (Einstein 1917)

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

- Quantum-vacuum energy (Zel'dovich 1968)

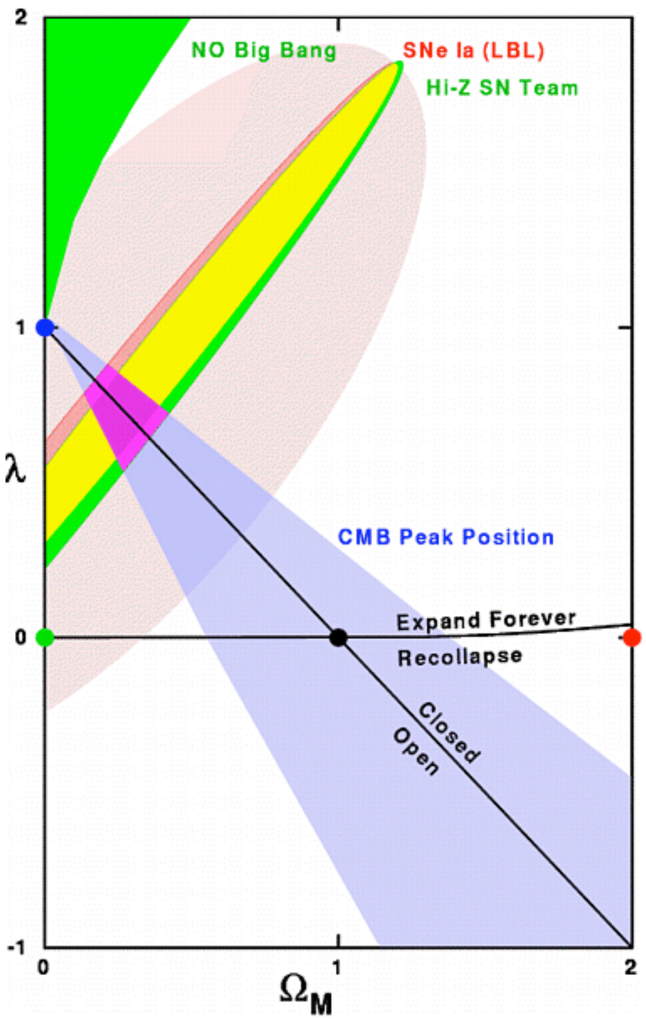
$$T_{ab}^{(\mathrm{vac})} = \frac{\Lambda}{8\pi} g_{ab} \qquad \rho_{\mathrm{vac}} = \frac{\Lambda}{8\pi} \qquad w = \frac{p}{\rho} = -1$$

- Quintessence - An unknown scalar field, ϕ

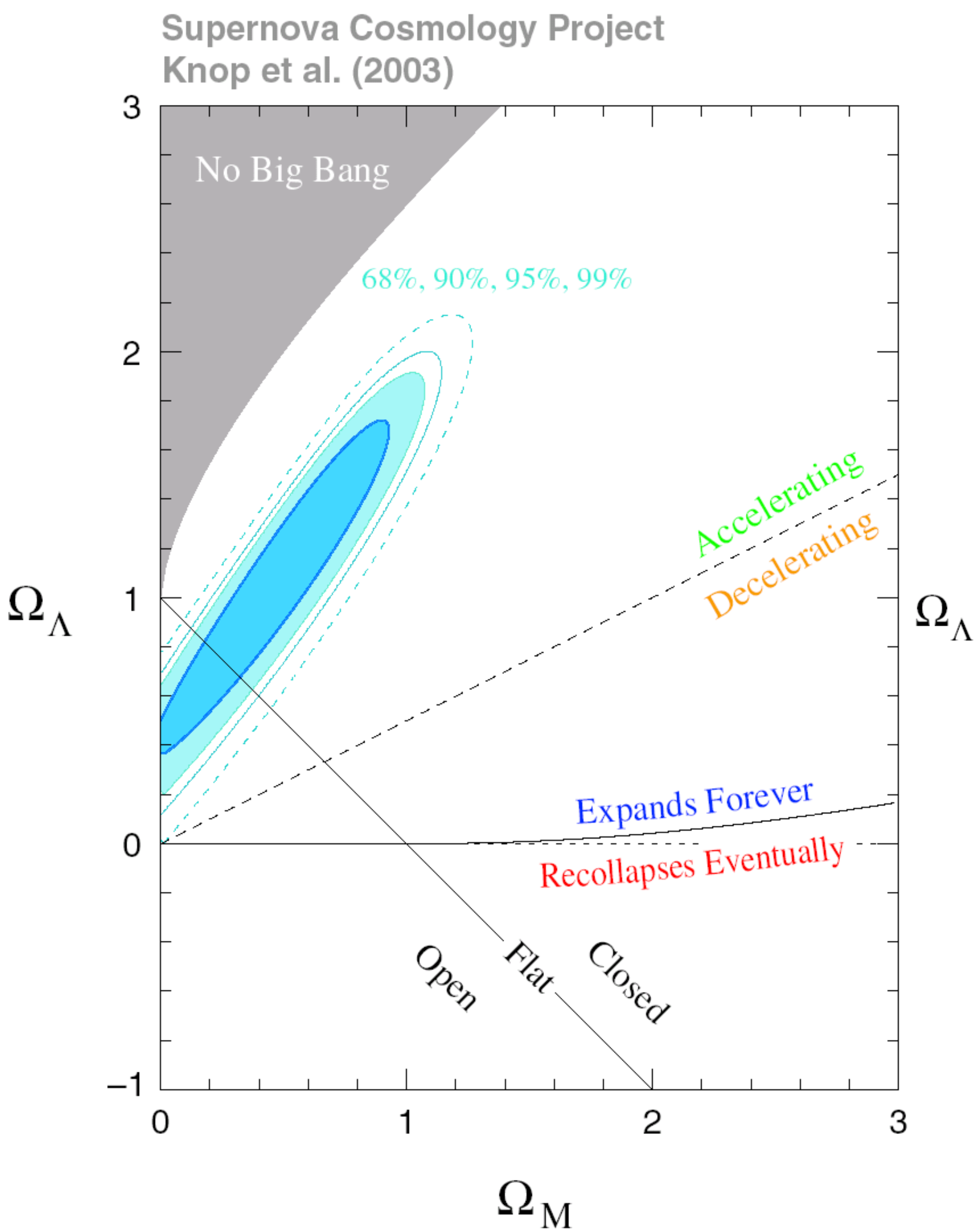
$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)},$$

- A sign that Einstein's gravity is wrong on large scales

A non-vanishing cosmological constant



- The simplest explanation of cosmic acceleration is that Einstein's cosmological constant is small but positive
- In this case fitting the SNa Ia Hubble diagram gives $0.8\ \Omega_m - 0.6\ \Omega_\Lambda \approx -0.2 \pm 0.1$
- As we will see, CMB anisotropies suggest that $\Omega_m + \Omega_\Lambda \approx 1.0$
- Therefore, one finds
  $\Omega_m \approx 0.2 - 0.3$
  $\Omega_\Lambda \approx 0.7 - 0.8$
- Additional datasets give consistent answers


Supernova Cosmology Project
Knop et al. (2003)
No Big Bang
68%, 90%, 95%, 99%
Accelerating
Decelerating
Expands Forever
Recollapses Eventually
Open
Flat
Closed
Ω_Λ
Ω_M
3
2
1
0
−1
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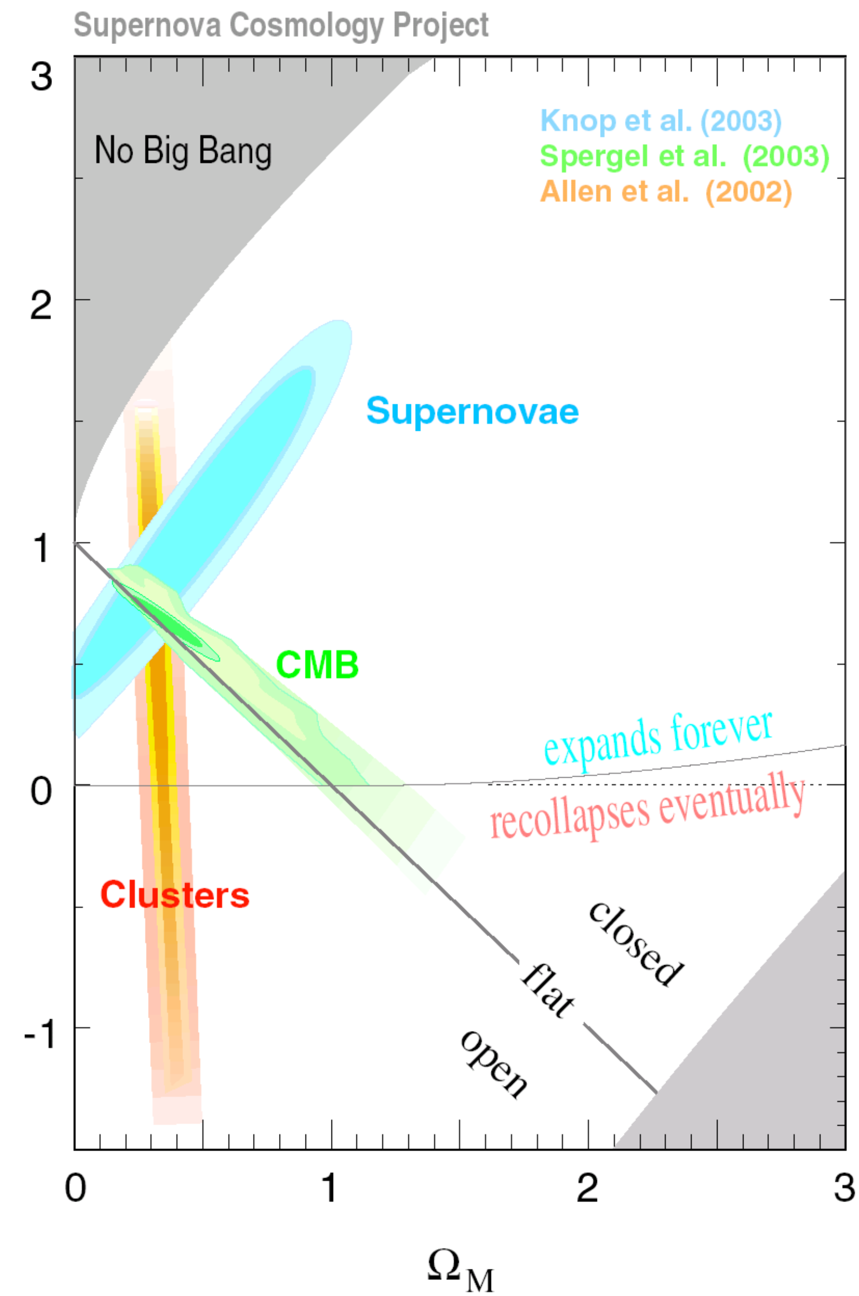
Supernova Cosmology Project
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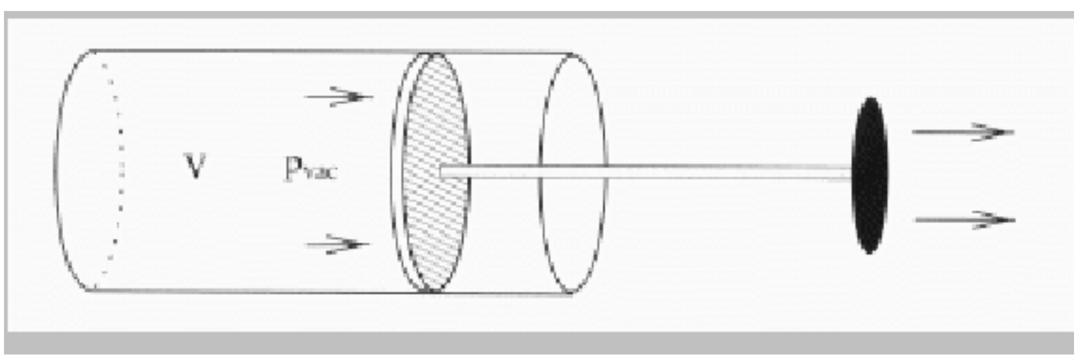
Spergel et al. (2003)
Allen et al. (2002)
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3

Modern interpretation of Λ

- Hermann Weyl attempted to link Λ to the quantum vacuum state
- In 1967, Yakov Zel'dovich noticed that if the vacuum state is a true ground state then all observers must agree on its form. But he realized that the only Lorentz invariant energy momentum tensor is the diagonal Minkowski tensor. Therefore, he proposed to move the Λ-term on the rhs of Einstein's field equations and to consider it as a source of energy-momentum which corresponds to a uniform sea of vacuum energy
- This corresponds to a fluid with $p=-\rho c^2$
- This can be seen from classical thermodynamics. The work done by a change in volume dV is equal to -pdV but the amount of energy in a box of vacuum energy increases when dV>0. Therefore p has to be negative.



Zel'dovich calculation

Explicitly the stress energy tensor for a fluid in its rest frame is

$$T_{\mu\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} \quad (1)$$

After a Lorentz boost in the x-direction at velocity $v = \beta c$ we get

$$\begin{aligned} T'_{\mu\nu} &= \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} \gamma^2\rho c^2 + \gamma^2\beta^2 P & \gamma^2\beta(\rho c^2 + P) & 0 & 0 \\ \gamma^2\beta(\rho c^2 + P) & \gamma^2\beta^2\rho c^2 + \gamma^2 P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix} \end{aligned} \quad (2)$$

While it is definitely funny to have $\rho_{vac} \neq 0$, it would be even funnier if the stress-energy tensor of the vacuum was different in different inertial frames. So we require that $T'_{\mu\nu} = T_{\mu\nu}$. The tx component gives an equation

$$\gamma^2\beta(\rho c^2 + P) = 0 \quad (3)$$

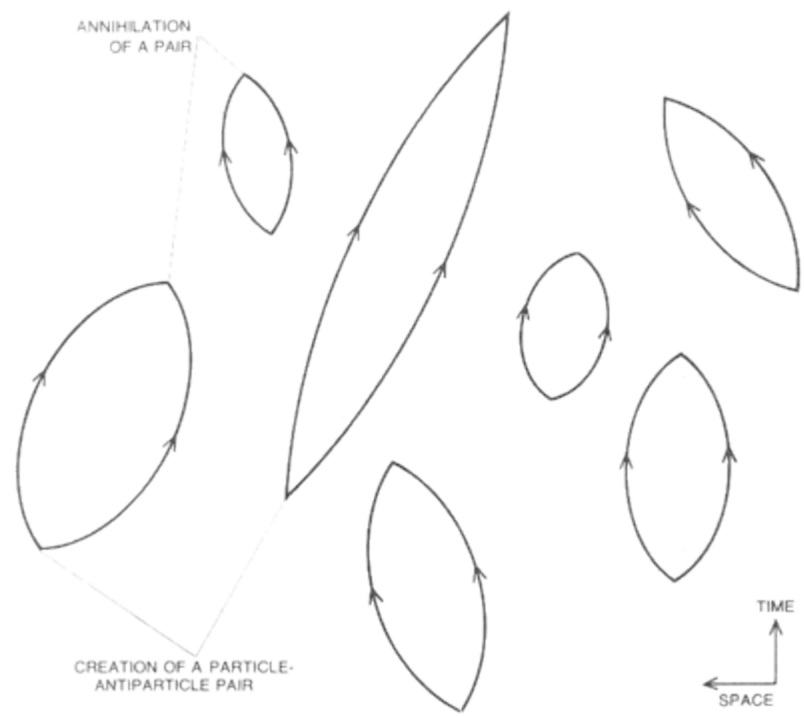
which requires that $P = -\rho c^2$. The tt and xx components are also invariant because $\gamma^2(1-\beta^2) = 1$.

Dicke coincidence argument
(why the vacuum energy should be zero)

- If SNa and CMB data are correct, then then vacuum density is approximately 75% of the total energy density today.
- At redshift 2 (nearly 10 Gyr ago for H_0=73 km/s/Mpc), the vacuum energy density was only 9% of the total
- 10 Gyr in the future, the vacuum energy density will be 96% of the total
- Why are we alive at the time when the vacuum density is undergoing its fairly rapid transition from a negligible fraction to the dominant fraction?
- This is an example of Anthropic reasoning

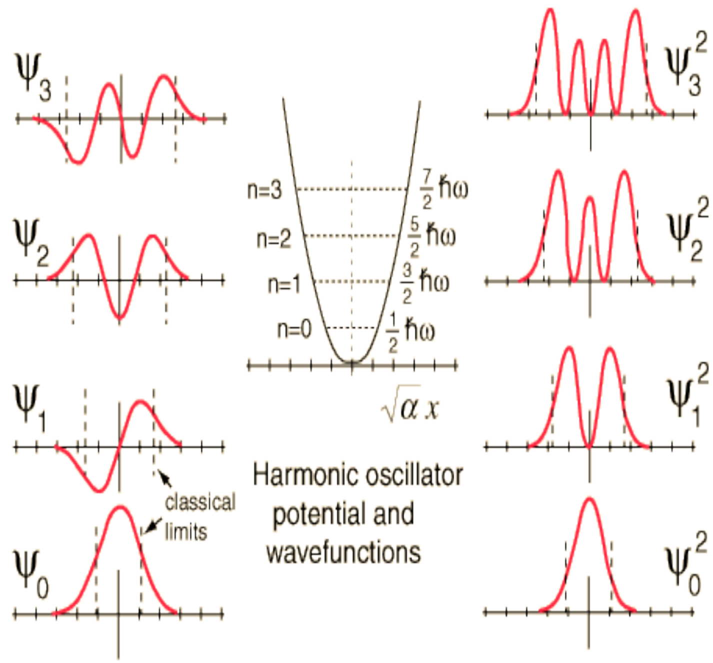
A dynamic vacuum state

- In the language of perturbative quantum field theory (Feynman diagrams), particle-antiparticle pairs ($\Delta E=2mc^2$) can be created from nothing as long as the energy is paid back in a time Δt which is short enough not to violate Heisenberg's uncertainty principle $\Delta E\ \Delta t > h/2\pi$
- This implies that the vacuum is not empty but it is teaming with virtual particles pairs
- Therefore empty space can have an energy density associated to it

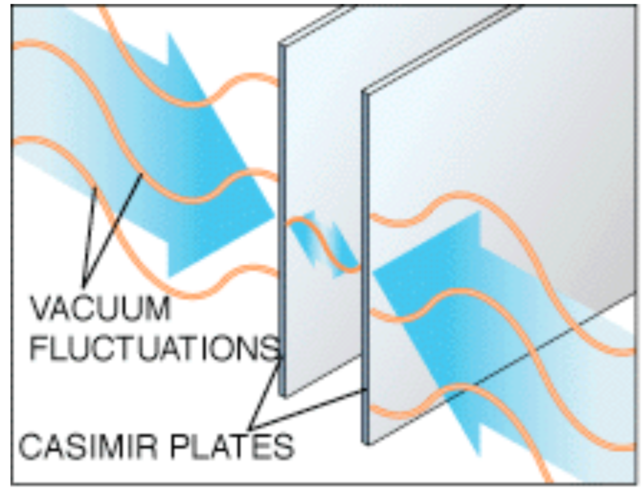


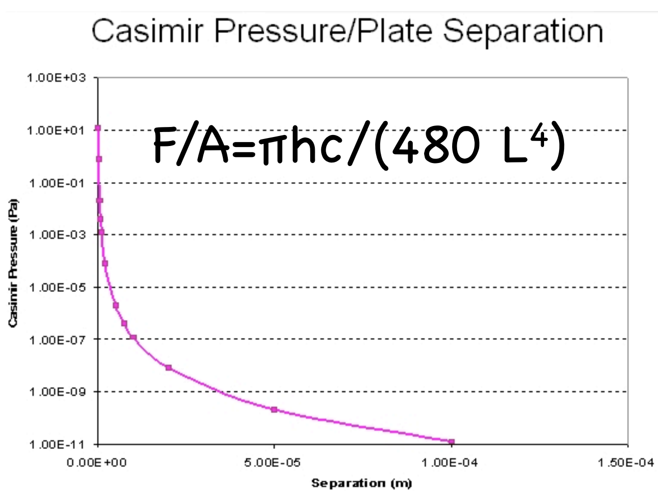
Zero-point energy (Nullpunktsenergie)

- Alternatively, vacuum energy can be seen as the sum of the zero-point energies of the quanta of the fields
- The minimum energy of an harmonic oscillator is $E_0=h\nu/2$, this is called the zero-point energy
- Quantum field theory can be regarded as a collection of infinitely many harmonic oscillators and therefore QFT predicts a non-zero vacuum energy
- Unfortunately we have no idea how to calculate it in a realistic way



Casimir effect

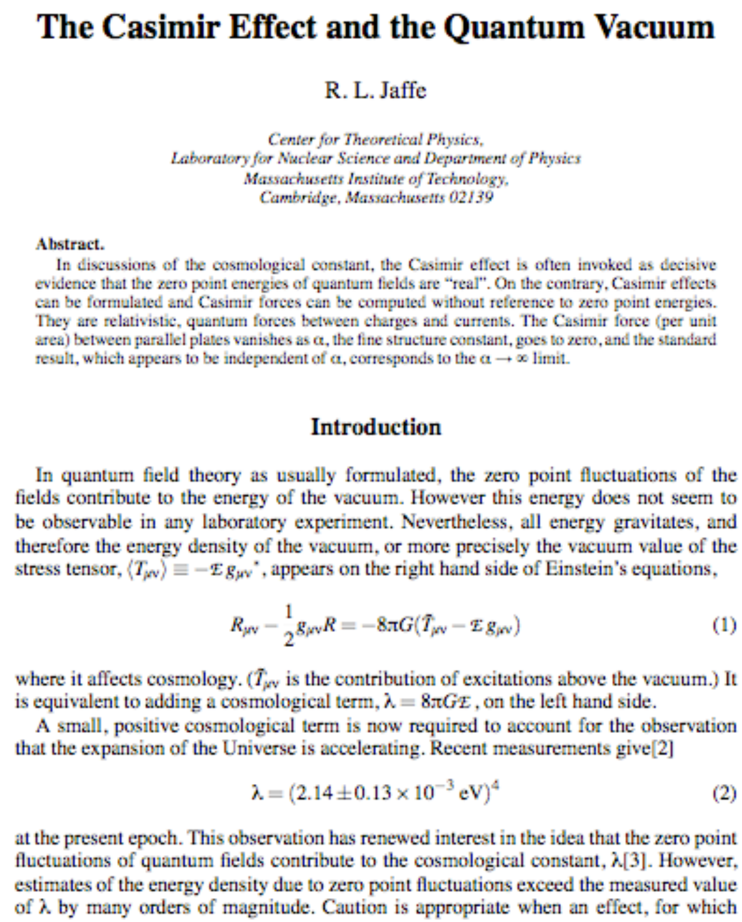




In 1948, Hendrik Casimir predicted that two close, parallel, UNCHARGED conducting plates should experience a small attractive force due to quantum vacuum fluctuations of the electromagnetic field. The tiny force has been first measured in 1996 by Steven Lamoreaux and by many others afterwards.

…no general consensus…

- Does the Casimir effect provide evidence of the "reality" of quantum fluctuations and zero-point energies?
- In 2005 R. L. Jaffe (MIT) showed that the Casimir effect can be computed without reference to zero-point energies
- In his calculation the effect originates from relativistic quantum forces between charges and currents
- Are zero-point energies of quantum fields real? Do they contribute to the cosmological constant?



The Casimir Effect and the Quantum Vacuum

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Abstract.
In discussions of the cosmological constant, the Casimir effect is often invoked as decisive evidence that the zero point energies of quantum fields are "real". On the contrary, Casimir effects can be formulated and Casimir forces can be computed without reference to zero point energies. They are relativistic, quantum forces between charges and currents. The Casimir force (per unit area) between parallel plates vanishes as α, the fine structure constant, goes to zero, and the standard result, which appears to be independent of α, corresponds to the $\alpha \to \infty$ limit.

Introduction

In quantum field theory as usually formulated, the zero point fluctuations of the fields contribute to the energy of the vacuum. However this energy does not seem to be observable in any laboratory experiment. Nevertheless, all energy gravitates, and therefore the energy density of the vacuum, or more precisely the vacuum value of the stress tensor, $\langle T_{\mu\nu} \rangle \equiv -\mathcal{E} g_{\mu\nu}$*, appears on the right hand side of Einstein's equations,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\tilde{T}_{\mu\nu} - \mathcal{E} g_{\mu\nu}) \quad (1)$$

where it affects cosmology. ($\tilde{T}_{\mu\nu}$ is the contribution of excitations above the vacuum.) It is equivalent to adding a cosmological term, $\lambda = 8\pi G\mathcal{E}$, on the left hand side.

A small, positive cosmological term is now required to account for the observation that the expansion of the Universe is accelerating. Recent measurements give[2]

$$\lambda = (2.14 \pm 0.13 \times 10^{-3}\ \mathrm{eV})^4 \quad (2)$$

at the present epoch. This observation has renewed interest in the idea that the zero point fluctuations of quantum fields contribute to the cosmological constant, λ[3]. However, estimates of the energy density due to zero point fluctuations exceed the measured value of λ by many orders of magnitude. Caution is appropriate when an effect, for which

The vacuum energy problem

- The measured value of Λ implies that the vacuum "mass" density is rather small ≈ 6 x 10^{-27} kg m^{-3} (the entire dark-energy content of the solar system equals the energy emitted by the Sun in 3 hours)
- If you naively sum up the zero-point energies of all the vibrational modes of a quantum field and assume that space-time is a continuum you get a divergent energy density (shorter wavelengths contribute more energy)
- If you admit that space-time might not be continuous at the Planck length and only consider modes with $\lambda > l_P$ you get an enormous but finite vacuum energy density ≈ 10^{96} kg m^{-3}
- If you also consider that fields are not free and that there are interactions between the modes you still find an answer which is tens of orders of magnitude away from the observed value
- For instance, if you adopt the minimal supersymmetric model and repeat the calculation you find that the vacuum energy is exactly zero. However, when the supersymmetry is broken (as it has to be today), you end up with a difference of nearly 60 orders of magnitudes.
- An unbearable amount of fine tuning is required to reconcile our present understanding in QFT with the observational data
- Note, however, that the naive QFT estimate agrees with observations if a cutoff at scales smaller than 1 mm is imposed

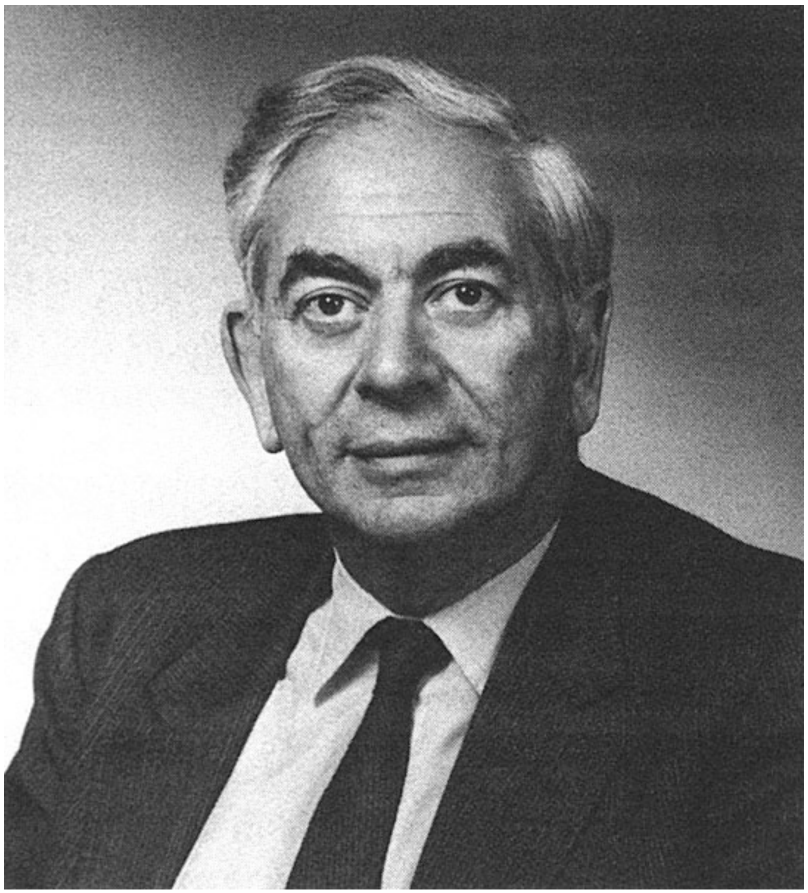
At the heart of the problem

- Physical phenomena in QFT are only determined by energy differences. Therefore diverging terms in the zero-point energy can be subtracted out. However, in general relativity is the total energy which gravitates and generates space-time curvature.
- Once again we need a unified treatment of gravity and quantum mechanics which is not available

Open questions

- Is the zero-point energy a physical quantity or just an artifact of our calculations?
- If it is physical, does it gravitate?

Dennis Sciama point of view



- "Even in its ground state, a quantum system possesses fluctuations and an associated zero-point energy, since otherwise the uncertainty principle would be violated. In particular the vacuum state of a quantum field has these properties. For example, the electric and magnetic fields in the electromagnetic vacuum are fluctuating quantities."
- "We now wish to comment on the unsolved problem of the relation between zero-point fluctuations and gravitation. If we ascribe an energy $h\nu / 2$ to each mode of the vacuum radiation field, then the total energy of the vacuum is infinite. It would clearly be inconsistent with the original assumption of a background Minkowski space-time to suppose that this energy produces gravitation in a manner controlled by Einstein's field equations of general relativity. It is also clear that the space-time of the real world approximates closely to the Minkowski state, at least on macroscopic scales. It thus appears that we must regularize the zero-point energy of the vacuum by subtracting it out according to some systematic prescription. At the same time, we would expect zero-point energy differences to gravitate. For example, the (negative) Casimir energy between two plane-parallel perfect conductors would be expected to gravitate; otherwise, the relativistic relation between a measured energy and gravitation would be lost."