Bachelorthesis

Constraining Binary Parameters based on the VLT-Flames survey

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1 Introduction

At the beginning, our Universe was composed almost exclusively of hydrogen and helium [de Mink, 2010]. The first stars burned hydrogen and helium in their interior. At the end of their life, they enriched the interstellar medium of heavier elements through powerful winds and explosions. These heavier elements include for example nitrogen, carbon, oxygen and iron. Using spectroscopy we are able to determine physical and chemical properties of stars, hence finally their age and evolutionary status.

Many stars form and evolve as single objects and the products of the internal nucleosynthesis can be mixed to the surface, e.g. if they have a high rotational velocity [Smiljanic et al., 2006]. Nevertheless, stars with a low rotational velocity and a high surface nitrogen abundance were found [Brott, 2011]. Trying to understand the characteristics of such stars, binarity may provide a possible solution. As many (over 70 % [Sana et al., 2012]) massive stars are found in binary systems, which are composed of two or more objects, their evolution can be drastically modified, compared to that of single stars. As a matter of fact, even if stars in a binary system are well separated and have long orbital periods (up to 1500 days), they can exchange mass due to the expansion happening as they evolve [Sana et al., 2012]. Stars on the main sequence can interact as well: it is believed that one third of the systems already interacts on the main sequence [de Mink et al., 2013a]. The interaction strongly affects the evolution of both stars, as well as their final fate [de Mink et al., 2013a].
This Bachelor thesis presents the results obtained by constraining binary parameters on the basis of a limited amount of observational information.

2 Theory

2.1 Evolution of Stars

In this thesis we focus on massive stars. The evolution of stars with masses exceeding $\sim$8 $M_\odot$ is not well understood [Langer, 2012]. After a star is formed and hydrogen is ignited in the core, the star holds a constant balance between the gravitational force which tries to collapse the star and the internal pressure gradient which acts in the opposite direction. The hydrogen is burned to helium and this reaction is efficient enough to take 90% of the star's lifetime [de Mink, 2010]. When the central hydrogen is exhausted, the internal pressure gradient decreases and the star starts to contract. During the contraction the pressure in the star's central region increases as well as its temperature. Hydrogen burning starts in a shell surrounding the core and, thanks to a further contraction, helium burning begins to produce carbon [Brott, 2011]. This process repeats several times, where progressively the main burning element moves on an outer shell and the core becomes "heavier" (i.e., atomic number of the main burning element increases). Massive stars with ten or more solar masses go through several of such stages until they have an onion structure of burning shells and form iron group elements in their core [de Mink, 2010]. Because iron has the highest binding energy per nucleon, nucleosynthesis can not provide energy any more and hence the core starts to contract under its own gravity. The increase in pressure and temperature leads to a break-up of iron into alpha particles which absorbs energy. The increase in density enables electron capture by protons which reduces the electron degeneracy pressure; due to the pressure loss both effects are accelerated further [Brott, 2011]. This leads to a runaway which causes the star to explode. However, there are several uncertainties on the last stages of massive star evolution.

2.2 Rotation

All stars rotate and the rotation rate is thought to have a major influence on the evolution of a star ([Langer et al., 2008]; [de Mink et al., 2013b]). The most relevant prediction is that rotation triggers internal transport of nuclear processed material from the core region to the outer layers. Rotational mixing is most important during the main sequence because after the hydrogen exhaustion the envelope expands and therefore slows down [Brott, 2011]. In massive stars hydrogen burns within the CNO-cycle. The reaction $^{14}N +^1 H \rightarrow^{15} O + \gamma$ has a very large half life time thus acting as a bottleneck, enriching the core region with nitrogen, which can now be transported to the surface by the rotational induced mixing [Brott, 2011]. However, the efficiency of such mixing process is not clear and it has been found that several observed massive stars do not match the theory's predictions, e.g. section 3.1 [de Mink, 2010]. This discovery questions the single star evolution theory.

2.3 Binarity

As said in Section 1 binary systems are a possible solution to explain the observation of enriched surface abundance for slow rotating stars. Stars in wide enough binary systems evolve like isolated ones, but for the close binaries the evolution may drastically change [Sana et al., 2012]. Binaries with periods up to 1500 days are able to exchange mass due to their expansion [Sana et al., 2012]. This happens when one of the stars fills its Roche lobe which is defined as the critical Roche surface [Hurley et al., 2002]. The Roche surface is an equipotential surface around the star because of the gravitational potential of both stars. The Lagrangian point L1 point is situated where the two equipotential surfaces of the two stars connect [Hurley et al., 2002]. When one star reaches this point, mass flows through it onto the companion [Hurley et al., 2002]. In other words, the Lagrangian point L1 can be found at the position where the gravitational forces of both stars and the centrifugal forces even out. Not only mass is transferred, but also angular momentum which can lead to mixing processes [Sana et al., 2012]. Tides play a further important role in close binaries (Hurley, Tout & Pols 2001): tides synchronize the spin of the star with the orbit and align the orbital rotation axis with the stars' rotation axis, as well as decreasing the orbit's eccentricity [Hurley et al., 2002].

2.4 The VLT-Flames Survey of Massive Stars

This Survey gathered spectra of over 470 stars in the Magellanic Clouds (Evans et al. 2006) **Quelle nicht fertig**, providing the chance to test theories of mixing processes in stars. In this thesis we focus on two clusters: NGC 2004 and N11 in the Large Magellanic Cloud (LMC). The LMC is located at a well known distance of 50 kiloparsecs [Pietrzyński et al., 2013] and hence one can determine the luminosity of the observed stars. This thesis therefore is based on the data of this particular survey. Amongst others the following properties are available which were chosen for this thesis because of their importance: Star number (for identification), Cluster (to know in which Cluster a star can be found), luminosity, temperature, projected rotational velocity, if its a SB1, mass, break-up velocity, surface nitrogen abundance and projected radial velocity amplitude. Due to the fact that their value of nitrogen and radial velocity amplitude is given, 37 SB-1 stars were selected for this thesis.

2.5 Inclination angle

The definition of the inclination angle can be seen in Fig. 1. It is the angle between the line of sight and the rotational axis of a star. Every observed star is, as said before, draped with this angle. Thus, it is important to know that every observed rotational velocity is just a minimum and the real one is almost certainly higher. The inclination angle is also important when observing the radial velocity amplitude of binaries which is the velocity a star has in its orbit around the center of mass (CM) in its system. Similar to the rotation it is unlikely that the line of sight is edge on with the orbital plane.

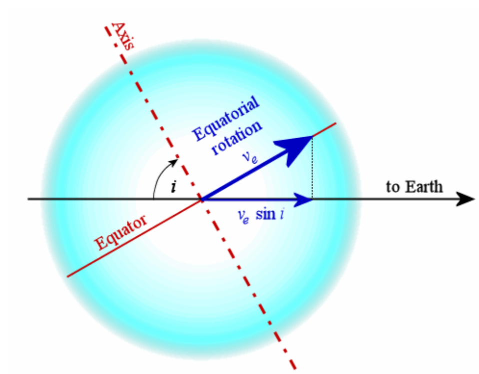

Figure 1: Representation of the inclination angle. The arrow (to Earth) indicates the line of sight to a given object. The Equator can either be the equator of a star, if we observe a rotational velocity or the orbital plane, if we observe the radial velocity amplitude for a binary star. The axis in both cases is the axis of rotation no matter if it is an orbital motion or just the rotation of the star around its own axis.

3 A short overview of the data

3.1 Hunter diagram

When taking a look at the Hunter diagram (Fig. 2), binaries with useful parameters, e.g. high nitrogen abundance can be found. As one can see, the diagram can be divided into Boxes. Box 3 and 5 are expected to contain most observed stars due to single star evolution because the number of simulated stars per bin is the highest. Box 4 has no observed stars neither is a significant amount predicted. In the other boxes we can thus find stars which do not fit with the predicted evolution of single or isolated stars and hence we have to find explanations for them. Actually the points in the diagram which are given by the VLT-Flames survey, especially the ones in Box 2, have encouraged this thesis.

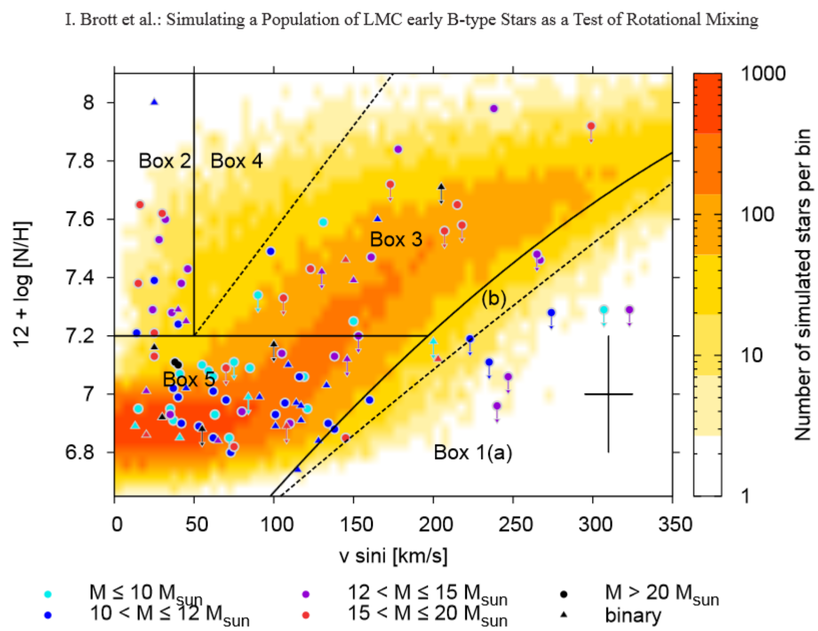

Figure 2: Hunter diagram which shows the projected rotational velocity against surface nitrogen abundance. The density plot in the background is given by the results of a simulation based on single star evolution theory and the color coding corresponds to the number of stars per pixel. Thus the path of red and orange pixel represents the single star evolution theory which means that every single star or isolated evolved star should be found there. The points represent observational data which are again color coded depending on their mass and indicated with a dot or a triangular if the observed star is in a binary system or not. The cross in the lower right corner shows the typical error on the observations (Brott et al. 2011).

Since the radial velocity amplitude is not given for all binaries that can be seen in Fig. 2 we made a Hunter diagram with the data (Table 1) given for this thesis (Fig. 3). We can find all stars that were given for both clusters with highlighted binaries. We have 33 stars for Cluster N11 with 17 Binaries and 29 stars for NGC 2004 with 20 Binaries. All binary systems in this plot are SB1 (single-lined binary). Now we divide the plot into the five areas given by the results of the simulation. The most interesting ones are, as said before, in Box 2 because the nitrogen abundance is high and the rotational velocity is low and Box 1 for the same reasons but vice versa. However, binaries in Box 3 might also be interesting because this Box is expected to contain stars which have evolved isolated. Thus, if we find close binaries with mass transfer in this Box one can question the idea of nitrogen enrichment due to binary interaction. Analogously for Box 5.

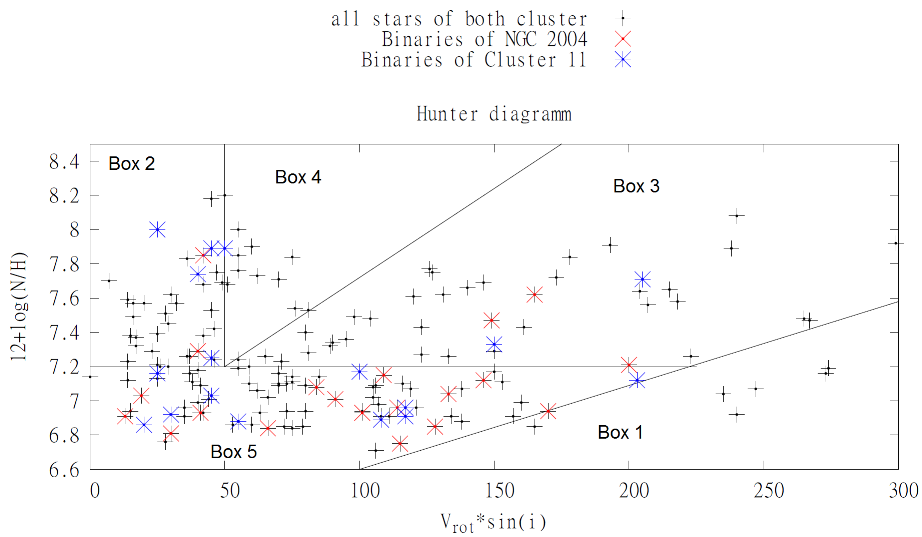

Figure 3: Stars of both clusters for which the nitrogen abundance and the rotational velocity is given. The Boxes are taken by the results of the simulated data which can be found in Fig. 2.

At first, we want to give an overview about the binary fraction in the different areas and about which impact the binaries in the different regions have. A reference to a Box will always relate to the ones in Fig. 3.

If we look at Box 2 we find 38 stars and 7 of them are SB1. So we have just a small binary fraction which leads to further questions. If the binaries are assumed to be the solution for high nitrogen abundance at the surface for slowly rotating stars as said in the theory, the question comes up how the other stars can be explained. Merged stars could be one possibility if we want to maintain a binary argumentation but of course there might be more explanations.

Box 4 contains 16 stars and one is a binary. Again the binary fraction is very low so the arguments given above are still valid.

Box 3 includes 4 binary stars out of 37 stars. This box is assumed to be filled with stars due to single star evolution; however, if we find a binary in that box which already has interacted with its companion one will have to question the idea of mixing due to binary interaction. Because of that this area will be kept into consideration in our analysis.

Box 5 contains 79 stars with 23 SB1 and will be absolutely negligible if no interactions take place. This is because in this box one can find stars that are consistent with the single star evolution and all slow rotating young stars should be found there.
Box 1 comprehends no binaries for this sample but a few single stars. One can therefore ask the question how these stars can rotate this fast without mixing nitrogen on the surface.
After this quick overview we now know that 11 SB-1 stars in our clusters show significant differences compared to the single star evolution theory.

3.2 Selection of stars from data

In order to find out which stars may be useful for further calculations, we use two criteria. The first one is the radial velocity amplitude. If this velocity amplitude is high (above 150 km/s) the stars are in a close system and hence mass transfer is possible. Finding a system with mass transfer one can test the idea of induced mixing processes due to binary interaction, because of the given surface nitrogen abundance. The second criteria is the surface nitrogen abundance. We will take a look at the properties of all binaries with a surface nitrogen above 7.2, which means that these stars can be found in Box 2,4,3 in the Hunter diagram (Fig. (3)).

Focus on radial velocity variations

In contrast to the rotational velocity the radial velocity amplitude is twice the amount of the orbit velocity. Both values are draped with the inclination angle i. If we take a look at our data (Table 1) we can see at first that the stars with high radial velocity amplitude are all located in the cluster NGC 2004. Because of that, we do not need to mention the cluster in the following. We extract the binaries of Table 1 that either feature a high radial velocity amplitude or a high nitrogen abundance. When pointing to the data we always refer to Table 1.

The highest radial velocity amplitude in the data is 260 km/s for star 47. Due to its extremely high radial velocity the stars may have interacted because the orbit has to be very close. So we take a look at the surface abundance of nitrogen. We find 7.04 (this number will always fit with the following definition of abundances: $12+\log(\frac{N}{H})$ and means that 7 is low and 8 is high) for the abundance relative to hydrogen which is relatively low if we take a look at the Hunter diagram Fig. 2. So either these stars have not yet interacted or this interaction did not lead to a mixing process.
The next star which seems to have a close system is 94 with a radial velocity amplitude of 200 km/s. We could now list the same arguments as above but again the nitrogen abundance is low with 7.08.
Star 50 also has a high radial velocity amplitude with 170 km/s. High in this case means above 150 km/s but nitrogen is just at an amount of 7.15.
Star 79 has a velocity of 160 km/s and an abundance of 7.62. This is therefore

cluster	star	Btype	v_{rot} [$\frac{km}{s}$]	Mass [$M_\odot$]	N	T_{eff}	Log(L)	$\triangle$ v_{rad} [$\frac{km}{s}$]	$\frac{v_{rot}}{v_{kep}}$
11	9	3	40	17	7.74	4.176	4.845	5	0.14
11	14	2	50	19	7.89	4.281	5.025	5	0.14
11	17	2.5	45	17	7.89	4.217	4.821	5	0.14
11	34	0.5	203	20	7.12	4.407	5.029	15	0.43
11	37	0	100	23	7.17	4.449	5.081	20	0.18
11	42	0	30	22	6.92	4.462	5.053	5	0.05
11	46	0.5	205	28	7.71	4.525	5.205	20	0.31
11	47	0	55	22	6.88	4.465	5.029	40	0.1
11	62	0.2	25	21	7.16	4.483	4.953	60	0.04
11	75	2	25	12	8	4.338	4.481	5	0.06
11	77	2	117	12	6.96	4.332	4.465	40	0.27
11	83	0.5	20	17	6.86	4.467	4.705	70	0.03
11	84	0.5	108	18	6.89	4.47	4.745	10	0.18
11	89	2	117	12	6.91	4.336	4.421	50	0.26
11	98	2	45	12	7.03	4.336	4.373	35	0.1
11	118	1.5	150	13	7.33	4.41	4.42	50	0.27
11	124	0.5	45	14	7.25	4.455	4.469	30	0.07
2004	3	5	42	20	7.85	4.16	5.042	5	0.16
2004	15	1.5	170	18	6.94	4.361	4.959	65	0.4
2004	20	1.5	149	18	7.47	4.361	4.907	85	0.34
2004	26	2	19	15	7.03	4.36	4.677	40	0.04
2004	29	1.5	30	14	6.81	4.364	4.649	20	0.07
2004	31	2	66	13	6.84	4.336	4.58	10	0.16
2004	41	2.5	101	12	6.93	4.311	4.428	75	0.24
2004	45	2	128	12	6.85	4.336	4.456	75	0.29
2004	47	2	133	12	7.04	4.336	4.416	260	0.3
2004	50	2.5	109	11	7.15	4.311	4.352	170	0.26
2004	54	2	114	12	6.96	4.336	4.392	50	0.25
2004	59	2	91	11	7.01	4.336	4.352	50	0.2
2004	78	2	115	11	6.75	4.336	4.26	90	0.25
2004	79	2	165	11	7.62	4.336	4.26	160	0.35
2004	88	2.5	200	10	7.21	4.311	4.176	15	0.45
2004	91	1.5	40	13	7.29	4.424	4.42	5	0.07
2004	94	2.5	84	10	7.08	4.311	4.156	200	0.19
2004	107	0.5	146	14	7.12	4.455	4.428	70	0.23
2004	108	2.5	13	10	6.91	4.354	4.208	15	0.03
2004	109	2.5	41	10	6.93	4.311	4.108	40	0.09

Table 1: This Table shows the most important properties of all binaries in Cluster N11 and NGC 2004 in the LMC which were observed by the VLT-flames survey. Column two gives the stars a number to identify them. N means the nitrogen abundance which has the same units as used in Fig. 2. T_{eff} is the logarithm of the effective temperature in Kelvin. L is the luminosity in Watt. The rotational velocity is named v_{rot} and the radial velocity amplitude v_{rad} , this notation will be used for the complete thesis. The last column $\frac{v}{v_{kep}}$ is a value which shows if the observed rotational velocity is near to the break-up velocity or not.

the first binary that might be interesting if the rotational velocity is for example low, because a close orbit implies a high radial velocity amplitude. But if we take a look at Table 1 one can see that the rotational velocity is relatively high with 165 km/s. So this binary might be interesting if one can find out if the stars have interacted, because in this case we would guess to have a higher nitrogen abundance at the surface.
The other stars do not seem to be very interesting because of the low radial velocity amplitude. However, the observed radial velocity amplitude is the lowest limit due to two effects. One is the inclination angle which means that we look at a certain angle at the system. If we do not luckily watch exactly onto the orbital plane, the observed radial velocity amplitude will always be lower than the real one. Due to the fact that we just plot our graphs with respect to the inclination angle and for close systems we will eliminate the inclination angle in our calculations which can be found in section 4.4, this is not that important. The second and more important one is that if we have a few data points of a given star, the radial velocity amplitude we obtain will be just a minimum because of the data epochs. We assume an orbit like the one shown in Figure (4) . The x-axis is scaled with a time and the y-axis with the radial velocity. If one observes at least one point which is on the horizontal line for y = 0.5 and one for y = -0.7 and no points outside the area between those two lines then the observed radial velocity amplitude will be the difference of the value y between these lines. If we get more points, e.g. more data epochs, the observed radial velocity amplitude will increase.

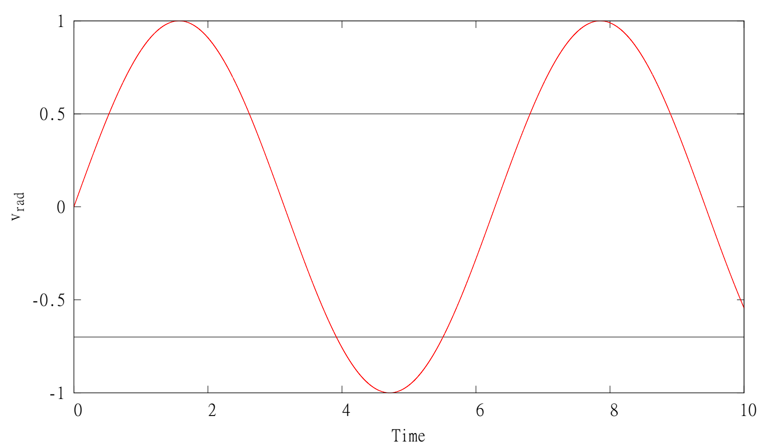

Figure 4: This figure shows the dependence of the radial velocity amplitude on the data epochs. The x axis is scaled with time and the y axis with radial velocity whereby the units are unimportant.

However, if the radial velocity amplitude is high it is less likely that this value will be much higher with more epochs.

Focus on high nitrogen abundance

Now we want to take a look at the binaries with high nitrogen abundances and check if there are any interesting parameters about the ones that have a high nitrogen abundances, e.g. high or low radial velocity amplitude. Clearly as one can see in Table 1 we now need to consider both clusters.
The first star that appears to be interesting is star 75 of cluster 11 with an abundance of 8. This is very high and if we take a look at the rotational velocity we will find a low one with 25 km/s. So the abundance is much too high when assuming a single star evolution. But also the radial velocity amplitude is low with around 5 km/s. This star is with high chance not interacting with its companion if we just consider the observed quantities. But we have a few possibilities: the real velocities are higher as the observed ones because of the effects mentioned before (inclination and epochs of the data points) or we have a pole-on view. If one could find out that both is not the case, this star questions the single star evolution theory.
The next interesting ones are 14 and 17 of cluster 11. They both have the same nitrogen abundance with 7.89 and nearly the same rotational velocity of about 50 km/s. Again we have a low radial velocity amplitude with 5 km/s, so the possibilities are the same as for star 75 in cluster 11.
With 7.85 the star 3 of NGC 2004 is the next interesting. Again the rotational and radial velocities are low (42 km/s and 5 km/s).
Star 9 of Cluster 11 has an abundance of 7.74 and it is slowly rotating with 40 km/s. The radial velocity amplitude is very low with 5 km/s.
Star 46 of the same cluster seems to follow the single star evolution with an abundance of 7.71 and a rotational velocity of about 205 km/s. Radial velocity is rather low with 5 km/s so there appears nothing to be special about this star.
The last one is star 20 of NGC 2004. It has an abundance of 7.47 and a rotation velocity of 149 km/s. So again single star evolution would work fine if the binary did not interact. However, the radial velocity amplitude is 85 km/s and therefore the stars might have interacted.
Star 118 in Cluster 11 has a nitrogen abundance of 7.33. The radial velocity amplitude is rather low with 50 km/s, but the rotational velocity is high with 150 km/s. The star therefore fits with single star evolution theory.
In Cluster 2004 the star 91 seems to be the next interesting with an abundance of 7.29. The rotation velocity is low with 40 km/s. However, the radial velocity amplitude is low with 5 km/s, too. Based on the fact that this star does not fit with the theory of single star evolution and surface nitrogen enrichment due to binary interaction, we might have a pole-on view.
With an abundance of 7.25, star 124 in Cluster 11 is the next one taken into account. The rotation velocity is low with 45 km/s just like the radial velocity amplitude with 30 km/s. This star has properties which can not be explained with single star evolution and hence this star will be used for further calculations.
The last star selected due to the surface nitrogen abundance is star 88 in cluster 2004. The nitrogen abundance is 7.21 and the rotational velocity is high with 200 km/s. This star fits therefore with single star evolution theory.

If we summarize these information, we have four possible close binaries with star

47, 50, 79, 94 of NGC 2004 or at least these are the ones where the possibility is high that they are close. Of course, due to the inclination angle and the lower limit for the observed radial velocity amplitudes other stars also might be close. Due to the abundance extraction we obtained cluster 11 star 9, 14, 17, 75, 124 and NGC 2004 star 3, 20 that appears to be interesting.

4 Calculations

4.1 Determing the radii of the stars

To calculate the star radii we use the Stefan-Boltzmann law:

$$L = 4\,\pi R^2 \sigma T^4$$

where L is the luminosity, σ is the Stefan-Boltzmann constant, R is the stellar radius and T is the effective temperature. Rewriting gives the star radius:

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

The resulting radii can be found in Table 2.

4.2 Calculating the semi-major axis

We assume the orbits to have a zero eccentricity. Now we want to estimate the semi-major axis of the primary which always means the more massive one of a given system (one could assume not to see the more massive one, but the calculations will be exactly the same, thus we can use one of the two possibilities). In order to do so we use Kepler's third law which is:

$$T^2 = \frac{4\pi^2}{G(M+m)}A^3 = \frac{4\pi^2}{G(M+m)}(R_1 + R_2)^3 \tag{1}$$

at which A is the mean distance between both stars, M is the mass of the primary, m the mass of the secondary and G the gravitational constant. We then exchange the distance with the semi-major axis R_1 of the primary and R_2 of the secondary. We want to obtain a range for the semi-major axis of the primary and hence we have to eliminate R_2. Thus, we need the definition of the center of mass (CM):

$$r_{cm} = \frac{R_1 M + R_2 m}{m + M} = 0 \tag{2}$$

We can set r_{cm} to zero because we take the CM as the origin of the coordinate system. Due to the circular orbits the stars are always on the opposite site of the CM, thus we do not need the vector notation. We obtain:

$$\mid R_2 \mid = \mid -R_1 \mid \frac{M}{m}$$

Using the relation between R_2 and R_1 we can modify equation (1):

$$T^2 = \frac{4\pi^2}{G(M+m)}(R_1 + R_1 \frac{M}{m})^3 \tag{3}$$

Because we assume the orbits to be circles, we now can replace the orbital period T with the orbit velocity (which can be obtained by dividing the radial velocity amplitude with factor two) of the SB1 star. The equation is:

$$T = \frac{2\pi}{w} = \frac{2\pi R_1}{v_{orbit}} \tag{4}$$

We can now exchange the period T in equation (3) with the one given in equation (4). With a few transformations we find:

$$R_1 = \frac{m^3 G}{v_{orbit}^2 (M+m)^2} \tag{5}$$

The secondary is conform with the inequation:

$$0 < m \leq M \tag{6}$$

Now we want to get a range for R_1 hence we can use the following two cases:

- m $\rightarrow$ 0 thus q $\rightarrow$ 0
- m = M thus q = 1

whereby q means the mass ratio defined as q = $\frac{m}{M}$. Using these limits for the mass of the secondary we get a range for R_1:

$$0 < R_1 \leq \frac{GM}{4v_{orbit}^2} \tag{7}$$

The radial velocity amplitude is given in Table 1 and hence the orbit velocity. We know that the velocity is just a lower limit because of the effects mentioned (inclination and data epochs). Therefore, the range given in inequation (7) can decrease if one knows the radial velocity amplitude more accurately.
Calculating the range for R_1 leads to Table 2 whereby the limits for the semi-major axis R_1 are given in units of the star radius of the primary.

clust	star	v_{rad} [$\frac{km}{s}$]	Mass [$M_{\odot}$]	R_1 (q = 1) [R]	R' [$R_{\odot}$]	i_{min} [°]
11	9	5	17	820	39	8.0
11	14	5	19	1208	30	8.0
11	17	5	17	1018	32	8.0
11	34	15	20	251	17	25.5
11	37	20	23	186	15	10.4
11	42	5	22	3117	13	2.9
11	46	20	28	278	12	18.1
11	47	40	22	51	13	5.7
11	62	60	21	26	11	2.3
11	75	5	12	1856	12	3.4
11	77	40	12	29	12	15.7
11	83	70	17	19	9	1.7
11	84	10	18	943	9	10.4
11	89	50	12	20	12	15.1
11	98	35	12	42	11	5.7
11	118	50	13	30	8	15.7
11	124	30	14	105	7	4.0
2004	3	5	20	714	53	9.2
2004	15	65	18	11	19	23.6
2004	20	85	18	7	18	19.9
2004	26	40	15	32	14	2.3
2004	29	20	14	126	13	4.0
2004	31	10	13	444	14	9.2
2004	41	75	12	8	13	13.9
2004	45	75	12	8	12	16.9
2004	47	260	12	1	11	17.5
2004	50	170	11	2	12	15.1
2004	54	50	12	20	11	14.5
2004	59	50	11	20	11	11.5
2004	78	90	11	7	10	14.5
2004	79	160	11	2	10	20.5
2004	88	15	10	216	10	26.7
2004	91	5	13	3205	8	4.0
2004	94	200	10	1	10	11.0
2004	107	70	14	20	7	13.3
2004	108	15	10	253	8	1.7
2004	109	40	10	33	9	5.2

Table 2: This Table shows the upper limit for the semi-major axis R_1 for the primary. v_{rad} is short for the radial velocity amplitude as said before and will be used for the whole thesis. The minimal inclination angle can also be found in this Table in unit of degree.

4.3 Determing the distance between the binary stars

Now we want to know the distance between the two stars. To do so, we use the CM (equation (2)) which is always located in the center of both orbits due to the circular motion. Because of that the distance is always the sum of the radii from both orbits.

$$\mathrm{A} = \mid R_1 \mid + \mid R_2 \mid$$

We use the absolute values because we obtain a negative value for the semi-major axis of the secondary. To get the distance we hence need to add the absolute values of both semi-major axis. Inserting this in our equation for the distance A and keeping in mind equation (5) we derived earlier we now get:

$$A = \frac{Gm^2}{v_{orbit}^2(m+M)} \tag{8}$$

Now we want to get a range for the distance A between the stars as we did for R_1. In order to do so, we take the derivative of A and set it to zero:

$$\begin{gathered}\frac{\partial A}{\partial m} = \frac{2mGv_{orbit}^2(m+M) - v_{orbit}^2 Gm^2}{v_{orbit}^4(m+M)^2} = 0 \\ \Longleftrightarrow m(m+2M) = 0 \\ \rightarrow m_1 = 0 \land m_2 = -2M\end{gathered}$$

The solution m_2 does not make sense, thus we have a minimum for m = 0 because the distance A (equation 8) is then zero. The distance increases monotonously with increasing mass of the secondary and keeping in mind inequation (6) the upper limit is given for m = M. Therefore, we find the following range for the distance A between the stars:

$$0 < A \leqslant \frac{GM}{2v_{orbit}^2} \tag{9}$$

Because we are interested in the mass ratio $\mathrm{q} = \frac{m}{M}$ and its dependence on A and the mass range, we rearrange Equation (8) and exchange $m = qM$ obtaining an equation with grade 2:

$$q^2 - q\frac{Av_{orbit}^2}{GM} - \frac{Av_{orbit}^2}{GM} = 0$$

With the solution:

$$q_{1,2} = \frac{Av_{orbit}^2}{2GM} \pm \sqrt{(\frac{Av_{orbit}^2}{2GM})^2 + \frac{Av_{orbit}^2}{GM}} \tag{10}$$

The solution with minus can not be true because otherwise one could get a q which is smaller than zero. We can now plot this function and get a curve which connects the distance A with the mass ration and with respect to a given radial velocity.

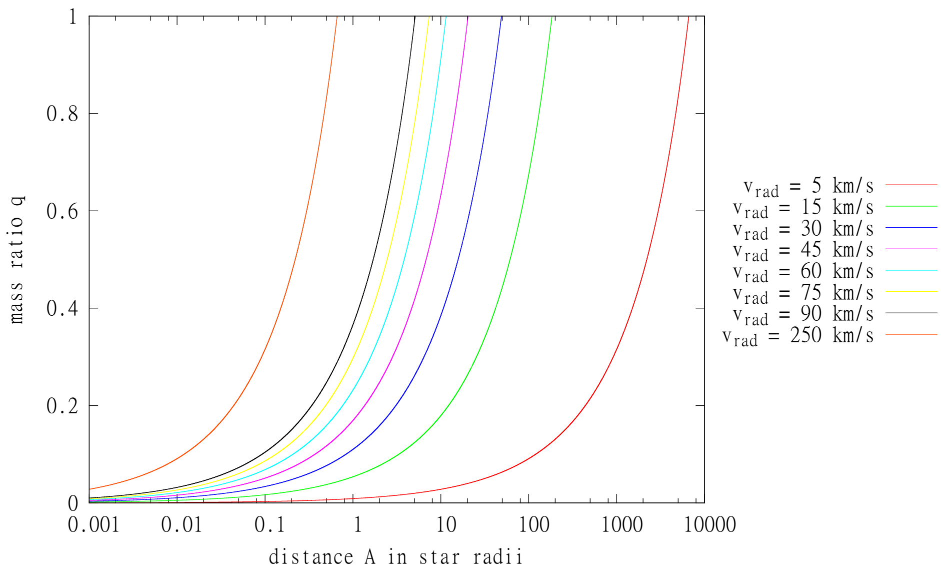

Figure 5: This graph shows the function q(a) (equation (10)) which is made for star 9 in Cluster 11 which has a mass of $17M_{\odot}$ and an observed radial velocity amplitude of 5 km/s. If we increase the radial velocity amplitude we can see how the function evolves to smaller distances A. The red line thus represents the observation; every other line is just made to get a better understanding of the evolution of this function and the used radial velocity amplitudes are not calculated

In Fig. 5 we can now see how the function evolves for higher radial velocity amplitudes from the right side to the left. For the used star 9 in cluster 11 we have an observed v_{rad} of 5 km/s, thus every configuration on the right side of the red line is impossible because the real radial velocity amplitude can just be higher than this value and for higher radial velocity amplitude the function moves to smaller distances.

4.4 Constraining the inclination angle

To get an upper limit for the radial velocity amplitude we take a look at the minimum inclination angle due to the break up velocity of the primary at its equator. We have given in our data (Table 1) $\frac{v}{v_{kep}}$. v is the observed rotational velocity and v_{kep} is the break-up velocity. Which means that the gravitational force F_{grav} equals the centrifugal force $F_{centrifugal}$. Whereby:

- F_{grav} = Mg
- $F_{centrifugal}$ = M ω^2 R'

- M the mass of the primary
- ω its angular velocity
- R' the primary star radius
- g = $\frac{GM}{R'^2}$
- $\omega = \frac{v_{kep}}{R'}$

$$Mg = M\omega^2 R'$$
$$\Leftrightarrow v_{kep} = \sqrt{g * R'}$$

Therefore, we find a relation for the inclination angle which leads to a minimum the system must have. Keeping in mind that the inclination angle is defined as the angle between the axis of the star and the line of sight according to section 2.5) we get the following equation:

$$sin(i) = \frac{v_{rot-obs}}{v_{kep}}$$

Since we assume the axis of rotation of the star is parallel to the axis of the orbit, we can take the same equation for our radial velocity.

$$sin(i) = \frac{v_{rad-obs}}{v_{rad-max}}$$

Now we have constrained the sin(i) in our data for the case of a close binary system which is useful to get the maximum radial velocity amplitude based on the observations, because one can now easily see which stars are unlikely close binaries. The velocity range we now have obtained is just a lower limit which depends on our observations. If the observed radial velocity amplitude increases: e.g. more data epochs, then the range of our possible velocities will increase, because we just fixed the minimum inclination angle. Remarkable about Fig. 5 is the fact that for lower mass ratio the distance between the two stars decreases significantly. This has to have a limit.

5 Mass ratio dependence on period and radial velocity variation

Because a distance given in stellar radii is not that useful to find out if a binary is close or not, we exchange the x-axis in Fig. 5 with the period in days which will lead to the final plots of this thesis. One can say a binary is close if the period is less than roughly twelve days. To obtain the equations we want, we have to go back to Equation (1), the Kepler law whereby the index 1 always means the properties of the primary.

$$T_1^2 = \frac{4\pi^2}{G(M+m)} \cdot A^3 = \frac{4\pi^2}{G(M+m)} \cdot (R_1 + R_1 \frac{M}{m})^3$$

And the one for R_1 equation (5) as well as equation (2) for the CM:

$$T_1 = \frac{2\pi}{w} = \frac{2\pi R_1}{\frac{v_{orbit}}{2}}$$

Now we can insert the second one by eliminating A in the first one:

$$T_1 = \frac{2\pi G m^3}{v_{orbit}^3 (m+M)^2} = \frac{2\pi GM}{v_{orbit}^3} \cdot \frac{q^3}{(q+1)^2} \tag{11}$$

where we used $q = \frac{m}{M} \Leftrightarrow m = qM$

Since we want the period to be our x-axis we have to calculate our function q(T) numerical. We calculated 1000 points for this function with Equation (11). Then we plot these points vice versa which can be seen in Fig. (6). In this plot we added a few possible orbital velocities due to the range of the inclination angle to show how the function evolves.

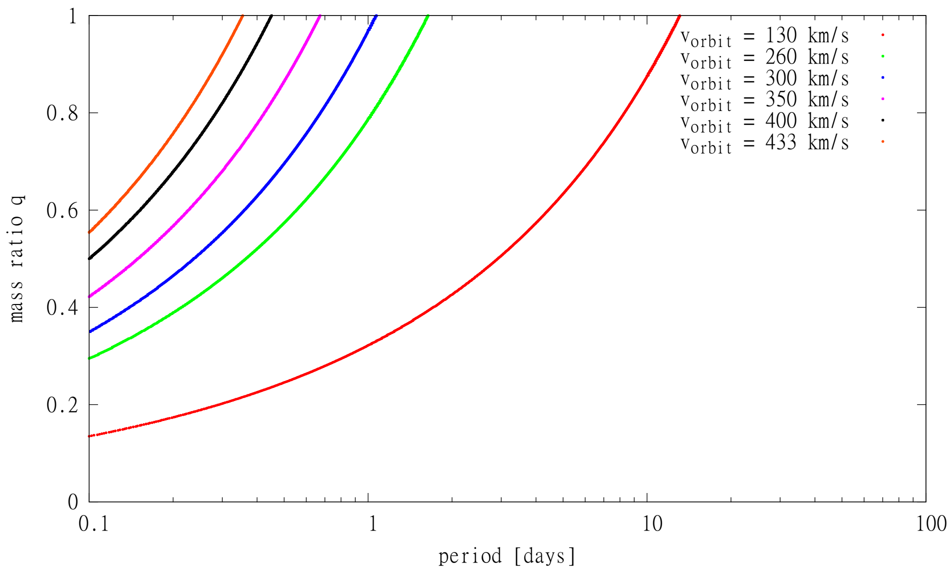

Figure 6: Star 47 in Cluster 2004 which has a mass of $12M_\odot$ and a radial velocity amplitude of 260 km/s. The x-axis is scaled logarithmic and in units of days. The y-axis is the mass ratio q. The colored lines show the function q(T) for different inclination angles and hence different orbit velocities.

One can neglect every correlations of parameters on the right hand side of the red line. This is due to the fact that the red line is based on the observational data and therefore the lowest limit of period and radial velocity of all. The maximum orbit velocity is given by eliminating the inclination angle. However, the red line might shift to shorter periods if the data epochs become more appropriate.

Now we want to find out for which configuration the system exchanges mass. We use the Eggleton approximation:

$$R_L = \frac{0,49A}{0,6 + q^{-2/3}log(1 + q^{1/3})} \tag{12}$$

with R_L the Roche lobe radius, A the separation and q the mass ratio. To easily see when a system has mass transfer, we want to find a away to implement this approximation in our plots. We divide the Roche lobe radius with the star radius and exchange A with equation (8) where we replace m by qM:

$$\frac{R_L}{R_{star}} = \frac{0,49GMq^2}{[0,6 + q^{-2/3}log(1 + q^{1/3})]v_{orbit}^2(q + 1)R_{star}} \tag{13}$$

with R_{star} the star radius, M its mass, v_{orbit} the orbital velocity, G the gravitational constant and q again the mass ratio. If this fraction becomes smaller than one, then the configuration of the system will be impossible, because the radius of the primary can not be much bigger than the Roche lobe radius.

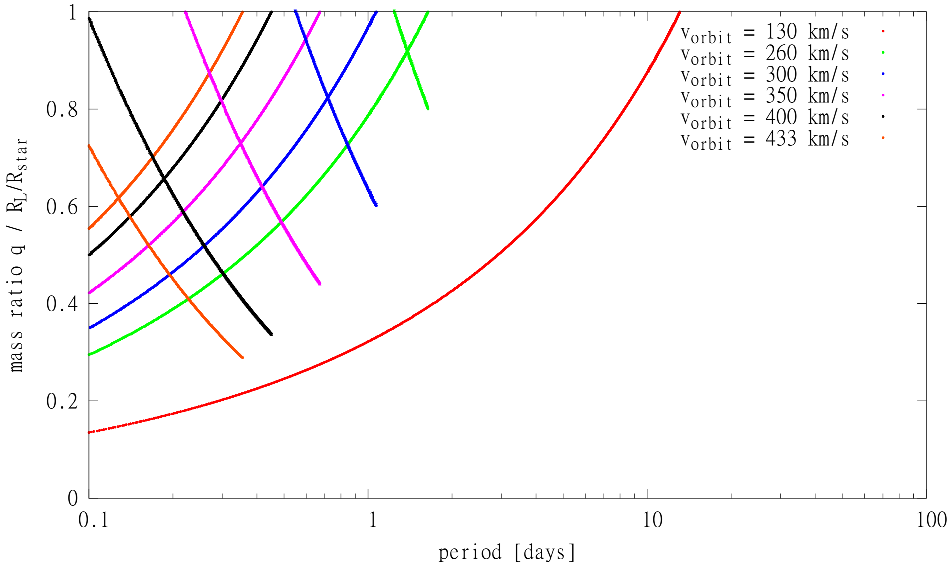

Figure 7: Caption same as for Fig. (6). Now the function of Equation (13) has been added. The colors again indicate the orbit velocity and hence two lines of the same color have the same orbit velocity.

The colors indicate the orbital velocity. We can see how Equation (13) evolves. At the point where the equation becomes one we now can find out which period and hence which mass ratio the system must have for mass exchange. We can now plot a function through these points and get a line of configurations which imply mass transfer.

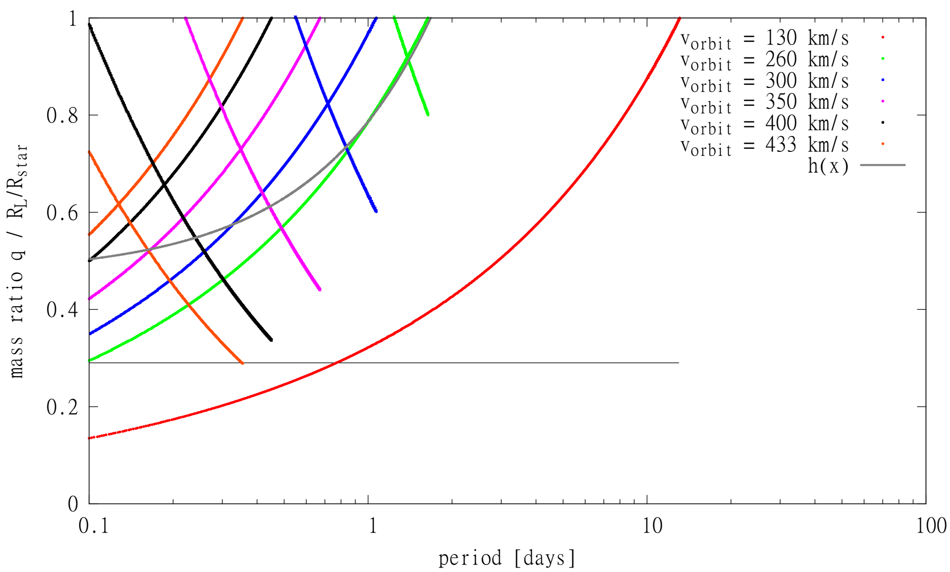

Figure 8: Caption same as for Fig. (7). h(x) is a fit which indicates for which configuration the system exchanges mass. The region above h(x) is impossible because for this case the radius of the primary exceeds the Roche lobe radius. The horizontal line at the bottom indicates that only a mass ratio below this line is possible if both stars of the binary systems are on the zero age main sequence.

The line at the bottom of Fig. (8) shows the maximum q the system can have due to the fact that all observed binaries in this sample are SB1. In order to observe a SB1 the secondary has to have a radius which is lower than roughly half the radius of the primary, because otherwise we would be able to observed the secondary with spectroscopy. The limit of half the radius was approximated by Luca Fossati (working in the AIfA, Bonn):

$$R_2 \lesssim \frac{R_1}{2} \tag{14}$$

We now use the mass radius relation for the ZAMS (zero age main sequence) [Demircan and Kahraman, 1991]:

$$R = 1.01M^{0.57} \tag{15}$$

Inserting equation (15) in equation (14) leads to:

$$q \lesssim 0.3 \tag{16}$$

The inequation gives a strong restriction for the system. However, it only works if both stars are on the ZAMS because otherwise we have to use other exponents for the mass-radius-relation. Since we can not be sure that this is the case, we plot it in every diagram (Section 5.1) to remind us about this possible limit.

Bound rotation

The last thing we want to calculate is a function q(T) which is based on the bound rotation assumption. This means that the primary star has the same angular velocity for the rotation and the orbital motion. Because of that, one can calculate a period based on the rotational velocity which is just the same as obtained in Equation (11).

$$T = \frac{2\pi R'}{v_{rot}} \tag{17}$$

whereby R' is the Radius of the primary and v_{rot} the rotational velocity. We can now eliminate T if we insert Equation (17) in Equation (11). We obtain:

$$\frac{(q+1)^2}{q^3} = \frac{R'(v_{orbit} * sin(i))^3}{GM(v_{rot} * sin(i))} \tag{18}$$

For every possible inclination angle we can now obtain the parameter q. Using function T(q) (Equation (11)) we get for every inclination angle the orbital period and hence we can plot a line in our diagrams which tells us exactly that the system has to have a configuration on this line if we have a bound rotation for the primary star.

Now we want to summarize what information can be obtained by our diagrams. First of all, we can find the function q(T) which was calculated numerical by using Equation (11). This function tells us for a given orbit velocity how the configuration of the system has to be. We then have the mass transfer function which was obtained by using Equation (13). This line therefore shows us which configurations are needed for the system to exchange mass and every configuration on top of this line is impossible, because in this case the radius of the star is much bigger than the Roche lobe radius. After this, we inserted the horizontal line which shows the upper limit for the mass ration in the case that both stars are on the main sequence. The last line added is the bound rotation line. If we assume the primary to have a bound rotation then the system has a configuration along this line.

5.1 Diagrams for stars extracted by their radial velocity amplitude

Now we take a look at the stars which were highlighted in the data extraction section.

Star 47 Cluster 2004

At first, we take a look at the functions q(T). Based on the observations it has a close orbit because it has a period lower than at least 13 days which can be seen in Fig. (9). The orange line in Fig. (9) shows the maximum possible orbit velocity due to the observed radial velocity amplitude and the eliminated inclination angle. The projected observed orbital velocity corresponds to the red line. The configuration of this system is therefore between the red and the orange lines. Both lines could shift to shorter periods as said before if the radial velocity amplitude observed increases. However, the shift can not assumed to be very big because the radial velocity amplitude with 260 km/s is high and the extreme case (orange line) is not very likely. Of course we still have to be careful. The configuration in reality can be outside the area between the lines because we have made the assumption of orbits with eccentricity zero which is an approximation for bound orbits.
Now we take a look at the mass transfer function. As said in the previous section every configuration on top of this line is impossible. Due to that the region of possible parameters is between the line of the mass transfer function and the observed red line obtained by the function q(T).
Taking a look at the horizontal line shows that the system can contain two main sequence stars. However, the region for this case can be found for very short periods with less than a day and is hence not that likely.
The last line is the function for bound rotation. As one can see this function can be found in the area of possible parameters and hence the primary can have a bound rotation.
The nitrogen abundance of this star is 7.04 which is low and the rotational velocity is 133 km/s. Taking a look at the Hunter diagram shows directly that

the star itself can be found in the region which can be explained with single star evolution. The diagram shows that mass transfer is possible, but has no high possibility.

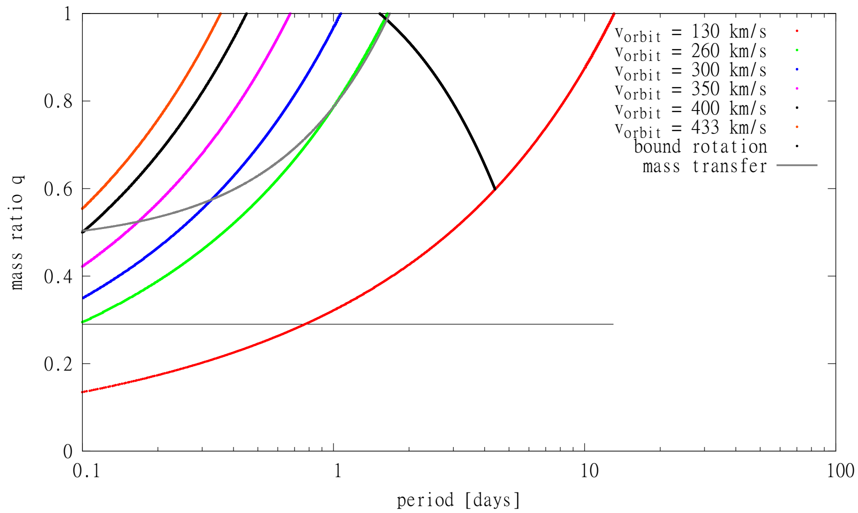

Figure 9: Star 47 in Cluster 2004 which has a mass of $12M_{\odot}$ and a radial velocity amplitude of 260 km/s. The x-axis is scaled logarithmic and in units of days. The y-axis is the mass ratio q. The horizontal line at the bottom indicates the maximum mass ratio the system can have if both stars are on the main sequence. The black line, named bound rotation, gives all possible configurations in case that the primary has a bound rotation. The grey line indicates the configurations for mass transfer and in addition every configuration on top of this line is impossible. The colored lines show the function q(T) obtained by inverting Equation (11).

Star 50 Cluster 2004

Star 50 has N = 7.15 and v_{rot} = 109 km/s. This star is in the same box of the Hunter diagram as the one before. Therefore, all arguments given above are still valid. We obtain a range of possible configurations between the mass transfer line and the red line which is based on the observed projected orbital velocity. Due to that, the maximum orbital velocity we can obtain based on the inclination angle is impossible as we can see, because it is located outside the are of possible configurations. If the system has a configuration between the horizontal line and the red line, the system will contain two mains sequence stars. If this is the case the primary can not rotate bound and vice versa.

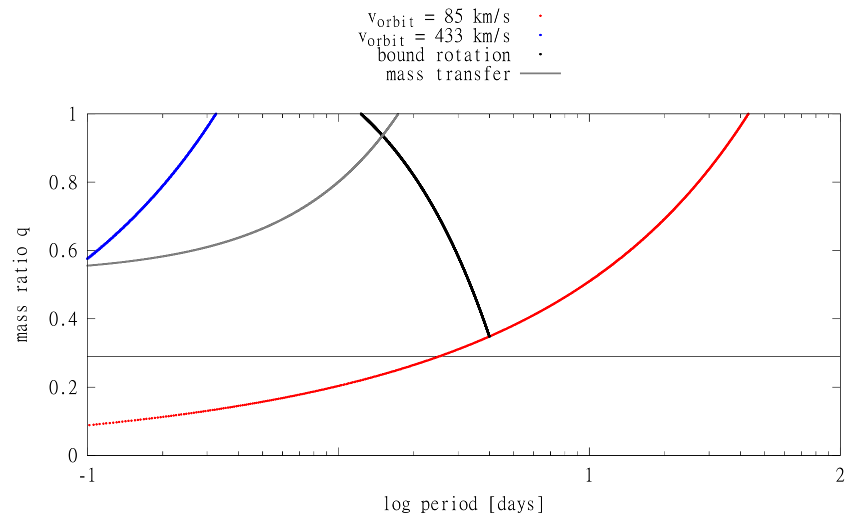

Figure 10: Star 50 in Cluster 04 which has a mass of $11M_{\odot}$ and a radius of 11.9 $R_{\odot}$. In addition all explanations that can be found in the caption of Fig. (9) are also valid for this star.

Star 79 Cluster 2004

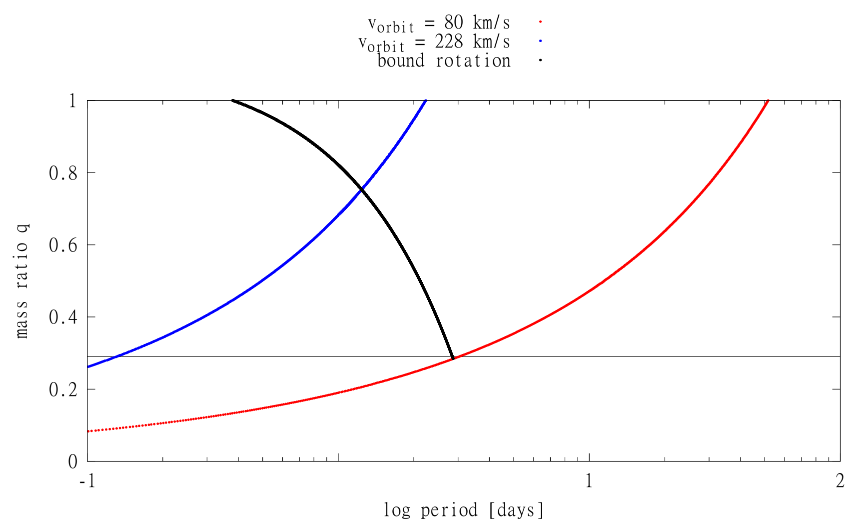

Figure 11: Star 79 in Cluster 04 which has a mass of $11M_{\odot}$ and a radius of 9.6 $R_{\odot}$. In addition all explanations that can be found in the caption of Fig. (9) are also valid for this star.

Star 79 has a high nitrogen abundance with 7.62 and a rotational velocity of 165 km/s. This star can be found in Box 3 in the Hunter plot Fig. (3). This area is the one which is expected for single star evolution so again although the binary seems to be close, one can not find that this leads to a disturbance of the abundances. The diagram shows that this binary does not have mass transfer. The primary can have a bound rotation because every configuration between the red and the blue line is possible but it can only rotate bound till a maximum of q for which one can find a number of 0.75 roughly. It is also possible that the binary systems contains two main sequence stars and the primary has a bound rotation (intercept point of the red and black line).

Star 94 Cluster 2004

Again a star with high observed radial velocity amplitude. The nitrogen abundance is low with 7.08 and the rotational velocity is 84 km/s. The same question as above appears. How are the chances to find close binaries without enriched nitrogen abundance if one expects them to go up with interaction? We can see that the primary can not have a bound rotation if the system exchanges mass. In this case it is also impossible that both stars are on the main sequence. Therefore, we get four possibilities for this system. The primary has a bound rotation, both stars are on the main sequence, the stars have mass exchange or none of these three options.

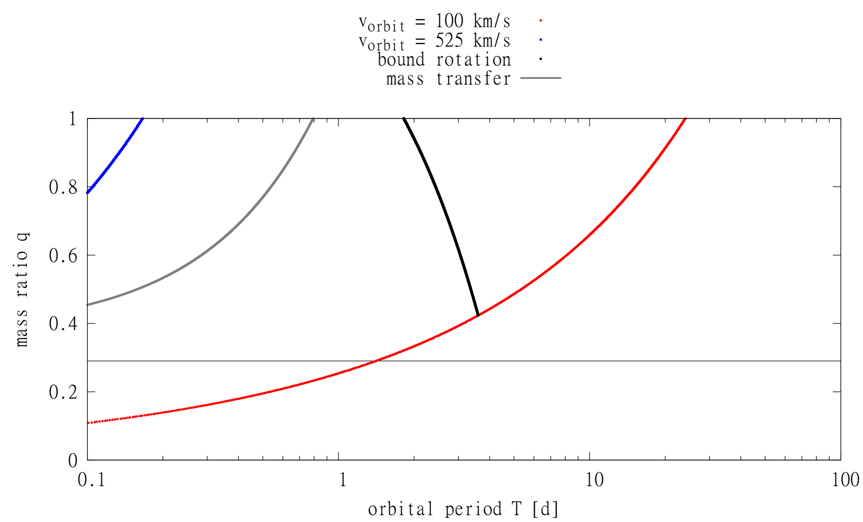

Figure 12: Star 94 in Cluster 04 which has a mass of $10M_{\odot}$ and a radius of 9.5 $R_{\odot}$. In addition all explanations that can be found in the caption of Fig. (9) are also valid for this star.

We found out that all binaries that are close have no special nitrogen abundance and therefore one can hardly use them to explain nitrogen enrichments due to binary interaction, but the plots also show that the configuration for mass transfer always implies an orbital period of less than two days. Because of that, the chance of interaction is low. Now we take a look at the stars with high abundance and low rotational velocity and normally vice versa but that does not make sense here because as we see in the Hunter diagram Fig. (3) there are no binary systems in Box 1.

5.2 Plots for high abundance but low rotational velocity

Star 9, 14, 17 of Cluster 11 and Star 3 of NGC 2004

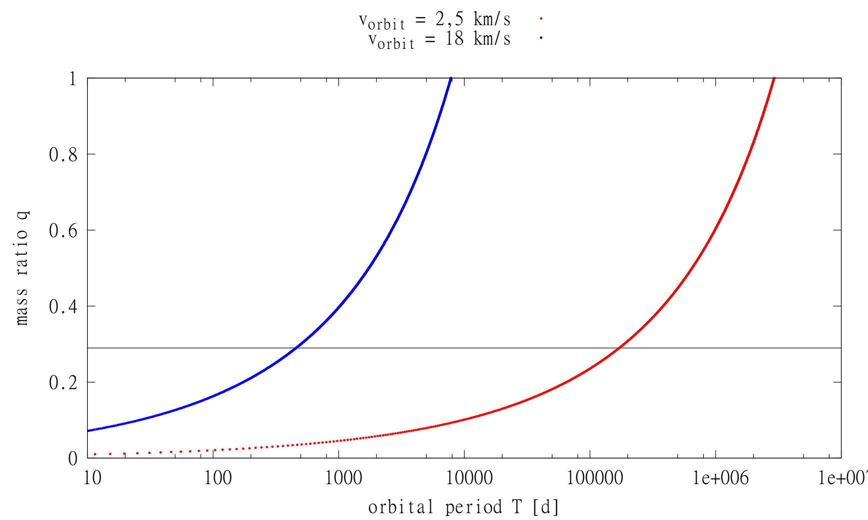

Figure 13: Star 14 in Cluster 11 which has a mass of 19,7$M_\odot$ and a radius of 29,8 $R_\odot$. In addition all explanations that can be found in the caption of Fig. (9) are also valid for these stars.

All four stars have very similar plots and hence we show one of them as an example. The inclination effect of those stars is very low so we have to have a huge change in our observed data to find a radial velocity amplitude high enough for bound rotation. The nitrogen abundance is roughly 7.8 for all four stars. The rotation velocity is, as mentioned before, low with around 45 km/s. Of course, due to the low observed radial velocity amplitude it is difficult to be sure that these systems can not be close binaries but it is not that likely for the following reason: These four stars have the highest nitrogen abundance among the stars used in this thesis and it is very unlikely that we have observed each of these stars with a pole-on view. Remarkable is that the radius of all four stars is very high compared to their mass around 30 $R_\odot$ for 14 and 17, nearly 40 $R_\odot$ for star 9 and over 50 $R_\odot$ for star 3 of cluster 2004. Maybe the rotation rate was higher and due to expansion the star slowed down. We refer to the Bachelor thesis of Marius Mürz how rotation velocities may evolve. If that is the case then the mixing process may has ended but the high abundance remains. With Section (5.3) we can see that these stars all have left the main sequence and hence the horizontal line can be ignored because it is based on this assumption.

Star 75 Cluster 11

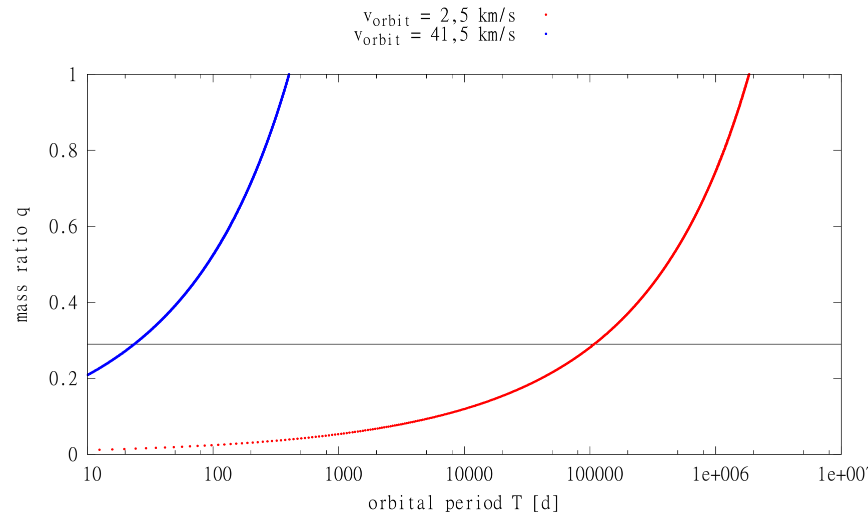

Figure 14: Star 75 in Cluster 11 which has a mass of $12M_{\odot}$ and a radius of 12.2 $R_{\odot}$. In addition all explanations that can be found in the caption of Fig. (9) are also valid for this star.

This star has the highest nitrogen abundance of all stars with a number of 8 relative to hydrogen. The rotational velocity is very low with 25 km/s. As one can see even if we eliminate the inclination angle the maximum radial velocity implies a very low chance to have a close system. The inclination elimination leads to a relatively high range of possible parameters. Therefore, if we just would have a slightly higher radial velocity amplitude observed, the maximum orbit velocity will increase strongly, which means that the chance of a close system increases strongly, too. With the data given the system can not be close and because the nitrogen abundance is high although the rotation velocity is low, this star leads to the answer that binarity can not explain the enrichment of nitrogen or at least it can not be the only option. The second option is that we have a pole-on view and hence, the rotation velocity is much higher.

Star 124 Cluster 11

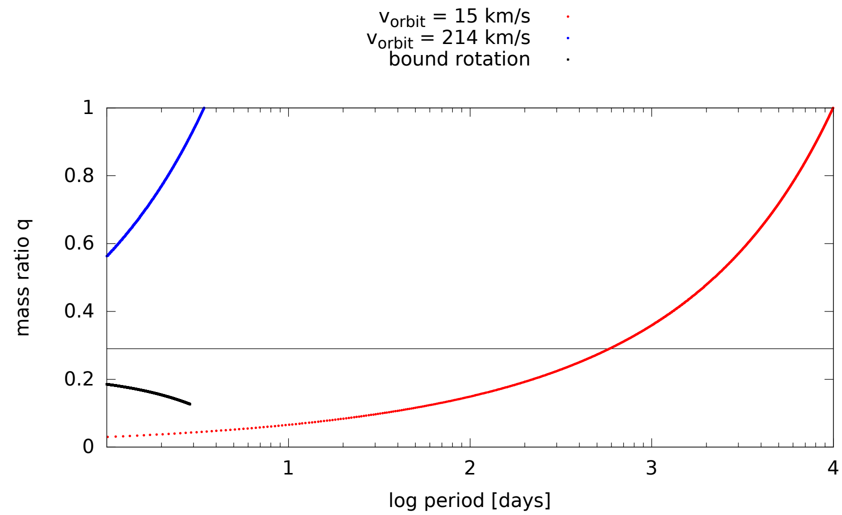

Figure 15: Star 124 in Cluster 11 which has a mass of $14M_{\odot}$ and a radius of 7 $R_{\odot}$. In addition all explanations that can be found in the caption of Fig. (9) are also valid for this star.

This star has a surface nitrogen abundance of 7.25 and a rotational velocity of 45 km/s. This star can therefore be found in Box 2 in the Hunter diagram Fig. (3). As one can see the area of possible parameters is very big but we can not find mass transfer. This star therefore supports strongly that the solution for an increase in surface nitrogen abundance can not only be found in binarity. If the primary has a bound rotation then both stars of the system are on the main sequence.

Star 20 of NGC 2004

This star can be found in Box 3 of the Hunter diagram Fig. (3). The radial velocity amplitude is not that high but as we can see, if we try to eliminate the inclination angle this star has a possibility to be in a close system. Nevertheless, this star is conform with single star evolution theory which means even if we find out that this binary is close, it will just show that close systems do not lead to an increase in the nitrogen abundance because we can not find mass transfer for any of the possible configurations.

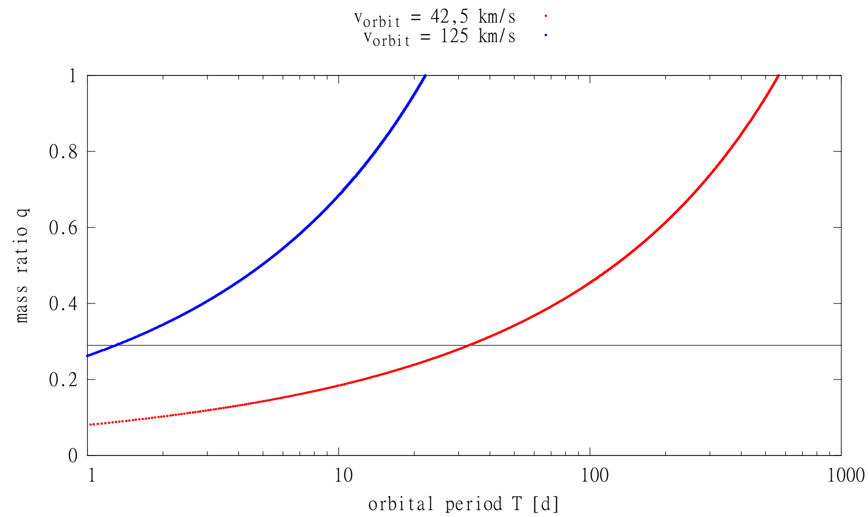

Figure 16: Star 20 in Cluster 2004 which has a mass of $18M_{\odot}$ and a radius of 18 $R_{\odot}$. In addition all explanations that can be found in the caption of Fig. (9) are also valid for this star.

5.3 HRD

The Herztsprung-Russel-diagram (HRD) is one of the most often used diagrams in astrophysics although it is roughly 100 years old. In this diagram two of the directly observable characteristics of stars are plotted against each other, the magnitude and the spectral type. The spectral type is directly related to the surface temperature and the magnitude can be translated to luminosity if the distance to the star is known. Thus one obtains the following diagram.

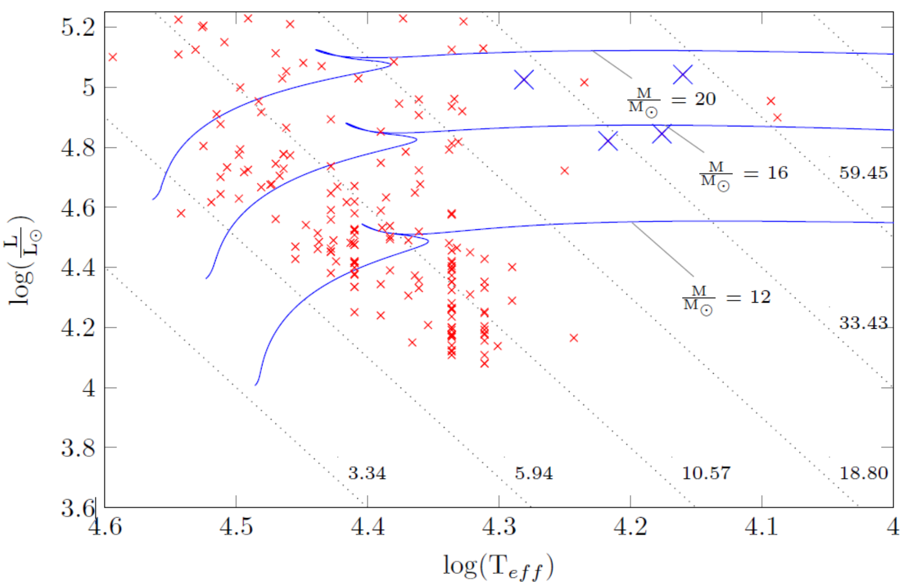

Figure 17: HRD of the stars of this thesis with lines of constant radii (black, dotted) with value $\frac{R}{R_\odot}$ and evolution tracks for 3 different masses with overshooting (tracks in blue)

The Blue tracks as said in the caption are evolutionary tracks and the blue crosses are the four binaries with high nitrogen, low rotation rate, low radial velocity amplitude and huge radii. With the blue tracks one can now easily identify the four stars. Obviously all of them have passed their MS-lifetime and are now on the way to helium core burning. This means that these stars have various opportunities to get their high nitrogen abundance because the evolution after the MS is not very well understood.

6 Conclusion

We have seen that the stars that seem to be in a close system have with high chance not interacted or, even if we assume the configuration leads to mass transfer, interaction does not force an enrichment of nitrogen abundance. All stars with high abundance and low rotation velocity are not in a close system and due to that the theory of binarity as a solution for stars with high abundance and slow rotation rates can not be proven with this work. However, we showed that the stars with high abundance in this sample have huge radii which leads to further considerations how these stars have enriched their surface abundance. Clearly they are no main sequence stars anymore which is shown with the HRD. The most interesting star (75 cluster 11) with high abundance and small radius is not conform with all accepted theories and hence a pole-on view seems to be a good explanation. Both Clusters together contain 37 SB1 stars thus this thesis can not be used as an analytical statistic. However, it is also unlikely that not even one star fits with the assumption of binarity as an answer to nitrogen enrichment if this assumption is correct. This work therefore indicates that other explanations for the enrichment of the surface nitrogen abundance have to be considered.

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Eidesstattliche Erklärung

Ich erkläre, dass ich meine Bachelor-Arbeit "Constraining Binary Parameters based on the VLT-Flames survey" selbstständig und ohne Benutzung anderer als der angegebenen Quellen und Hilfsmittel angefertigt habe und dass ich alle Stellen, die ich wörtlich oder sinngemäß aus Veröffentlichungen entnommen habe, als solche kenntlich gemacht habe. Die Arbeit hat bisher in gleicher oder ähnlicher Form oder auszugsweise noch keiner Prüfungsbehörde vorgelegen.

Bonn, den

(Name des Kandidaten)