BRUTUS

a brute force arbitrary-precision N-body code

"*What degree of accuracy is enough?*"

(Smith 1979)

"*How badly are we allowed to integrate?*"

(Heggie 1991)

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Sterrewacht Leiden

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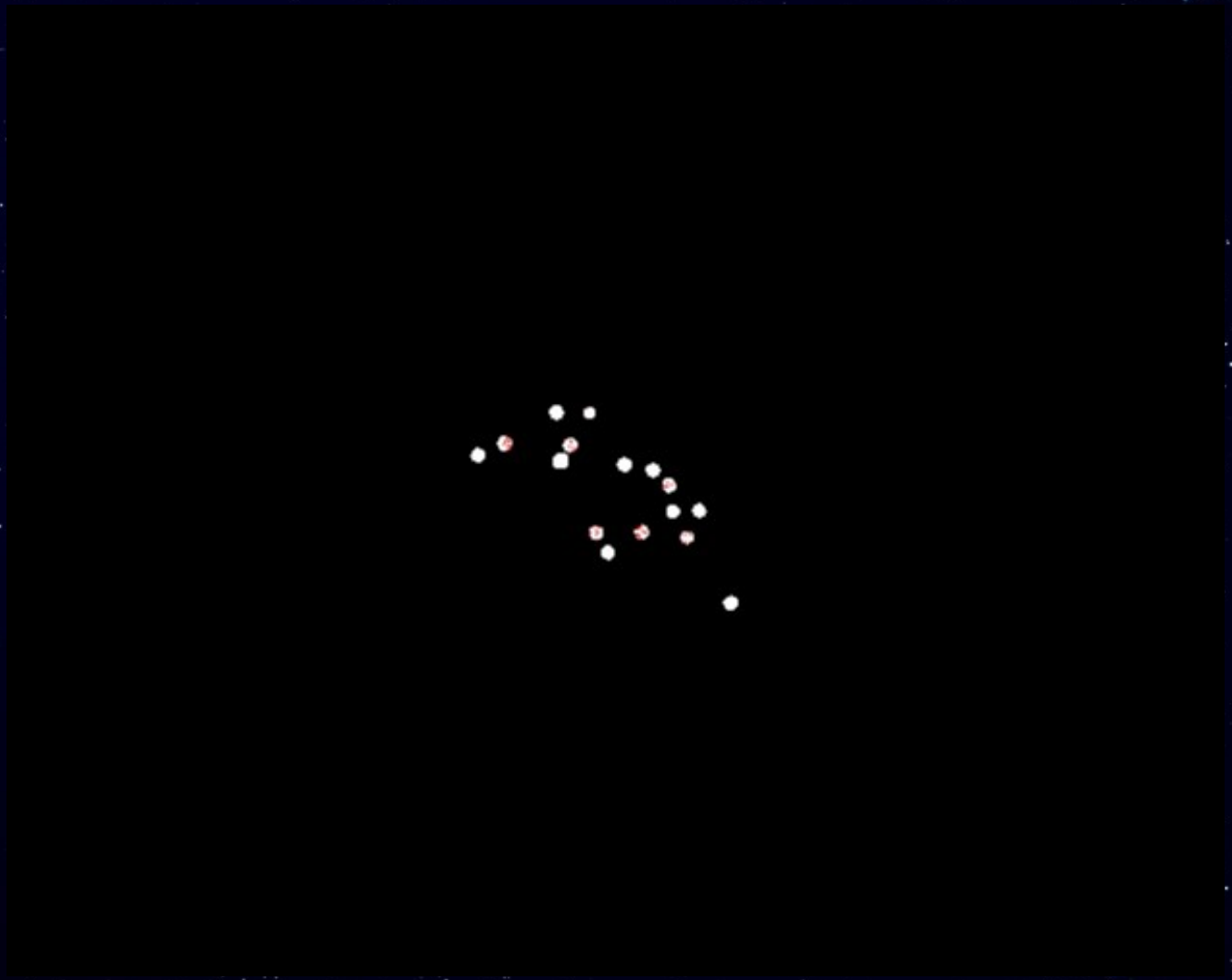
CASTLE Group

Supervisor:

Prof. Portegies Zwart

Part 1: Individual Accuracy



Part 1: Individual Accuracy

Introduction

- Phase-Space Distance, (Miller, 1964)

  between 2 simulations with the same initial conditions:

$$\delta = 0.5 \log_{10} 1/(6N) \sum (x_2 - x_1)^2 + (v_2 - v_1)^2$$

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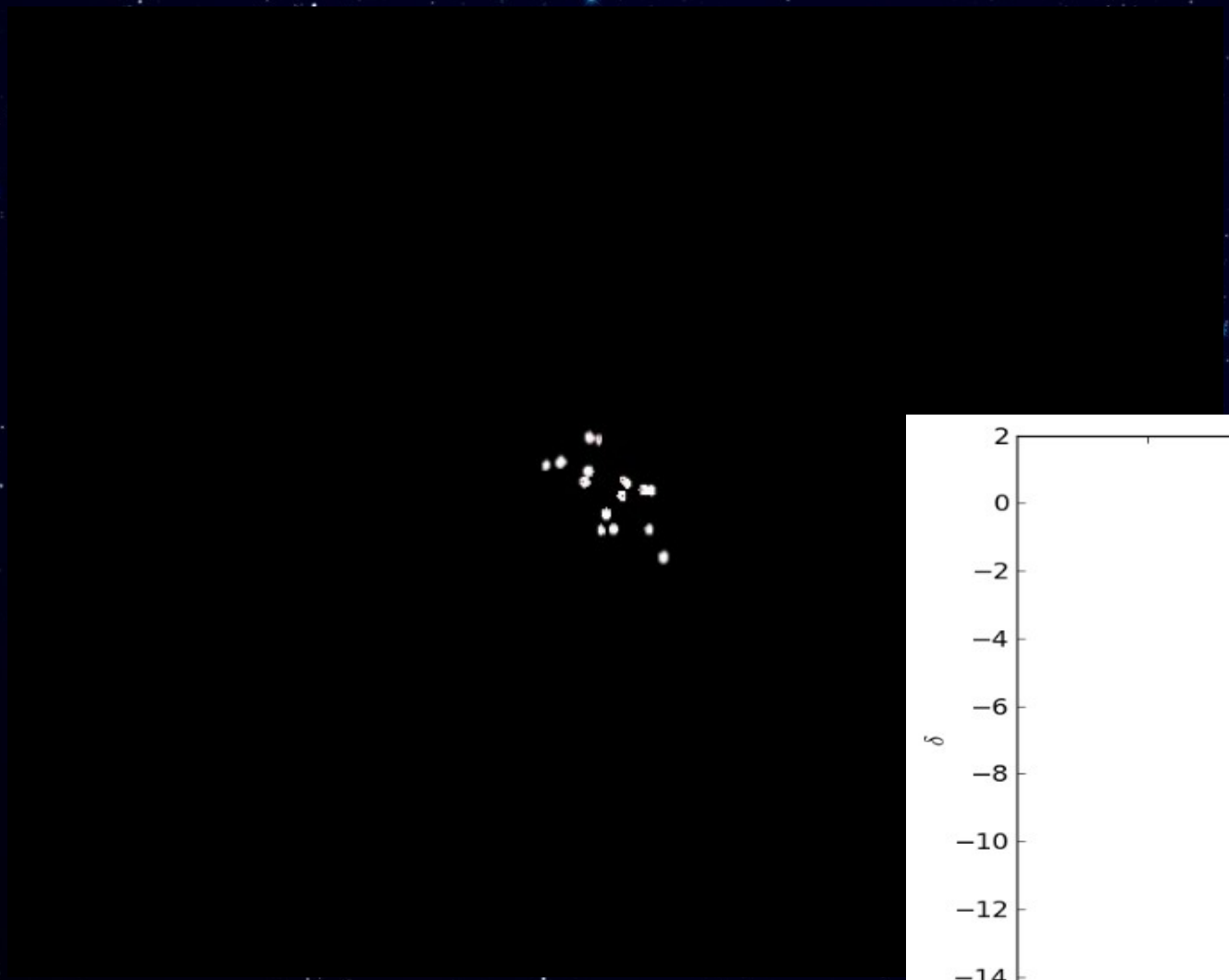
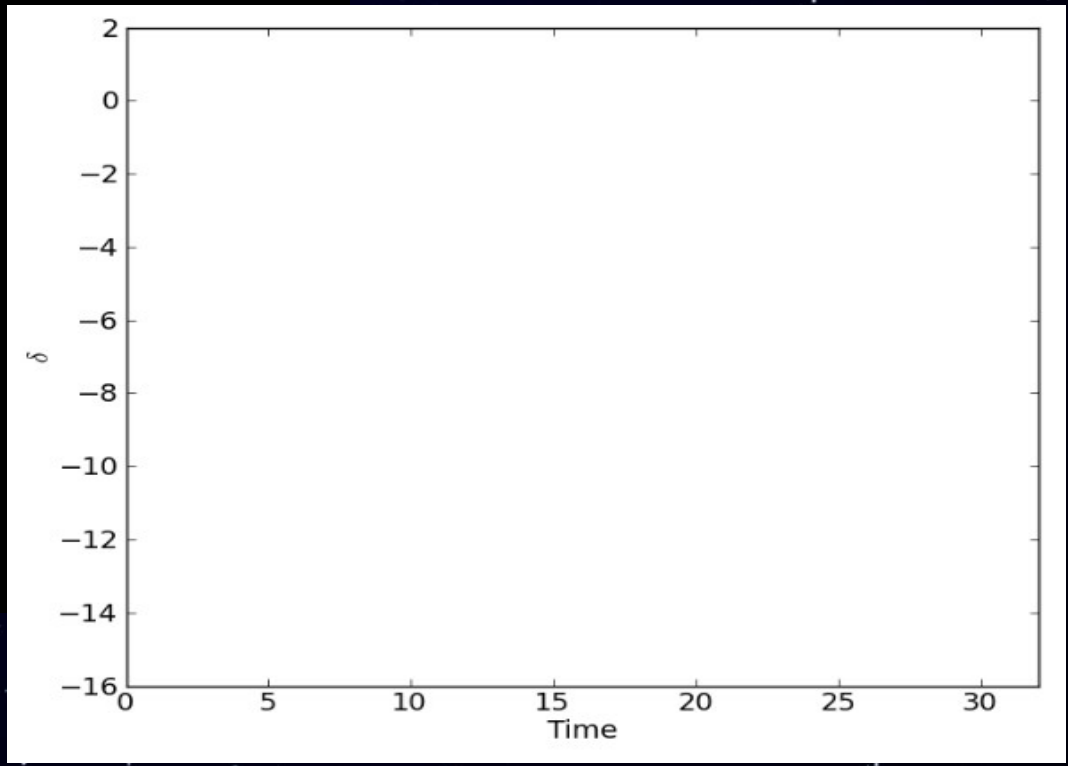
N=16 Plummer Experiment





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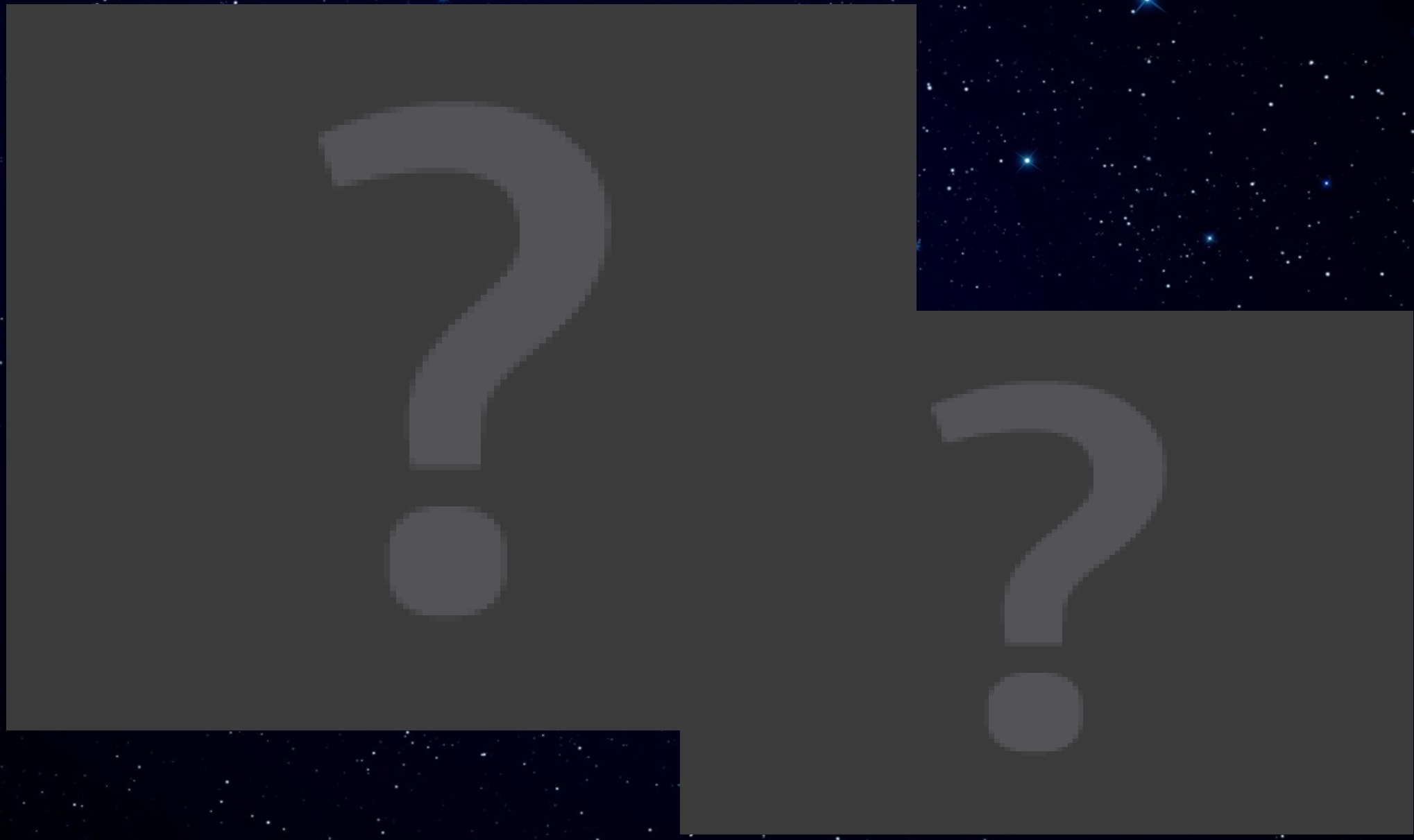


Introduction

- Two sources of error
  - Discretization error *(discrete time step size)*
  - Round-off error *(finite number of digits)*

- Exponential divergence (Goodman et al. 1993)
  - Divergence away from the true solution due to accumulation of errors
  - Time scale of the order a crossing time

BRUTUS

a brute force arbitrary-precision N-body code

- Two ingredients:
  - Bulirsch-Stoer method
    - Tolerance parameter, Tol
  - Arbitrary-Precision
    - Number of bits, N_{bits}

BRUTUS

a brute force arbitrary-precision N-body code

- Bulirsch-Stoer method
  - Leapfrog, 2nd order
  - Iterative integration
  - Extrapolation to zero time step size
  - Tolerance parameter, Tol (= 1e-6, 1e-8, 1e-10, ...)

Control discretization error

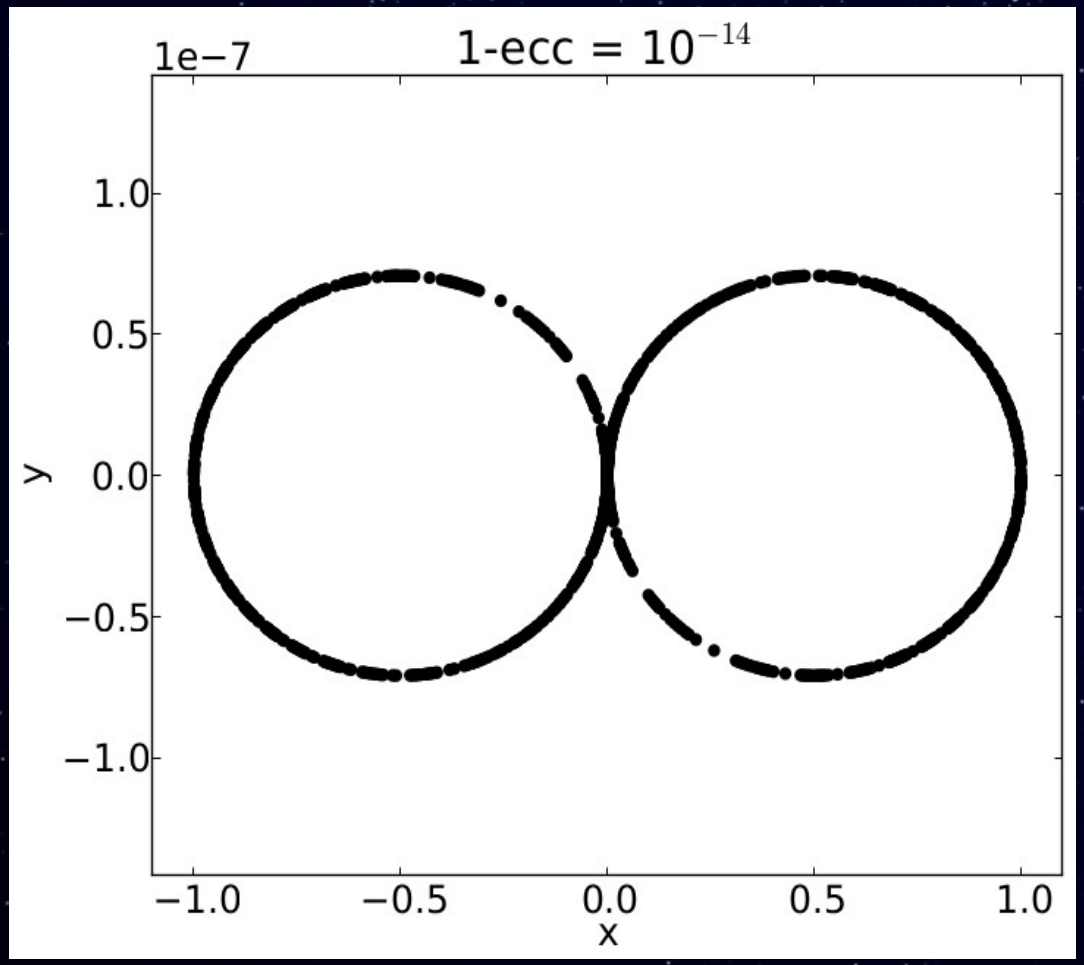
BRUTUS

a brute force arbitrary-precision N-body code

- Arbitrary-Precision
  - GMP, MPFR, MPFR C++
  - Number of bits, N_{bits} ($N_{digits} \sim N_{bits}/4$)
  - N_{bits} = 56, 64, 72, 80, 88, ...

Control round-off error

Brutus - Example

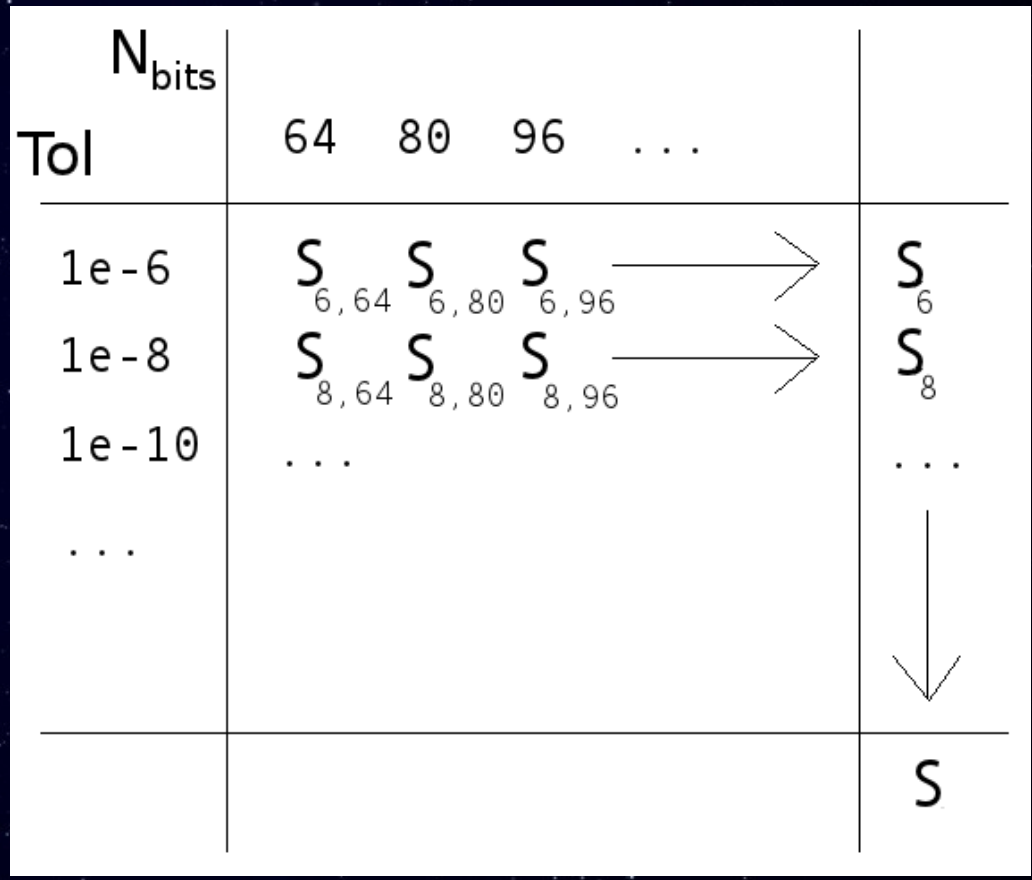


- N = 2, binary
- T_{sim} = 100 (P=2π)
- Tol = 1e-14
- N_{bits} = 88
- T_{cpu} = half hour
- dE = 3e-7

Method of Convergence

- Initial State

- Vary Tol, N_{bits} systematically
  - S(Tol, N_{bits})
- Convergence in N_{bits}, per Tol
  - S(Tol)
- Convergence in Tol
  - S



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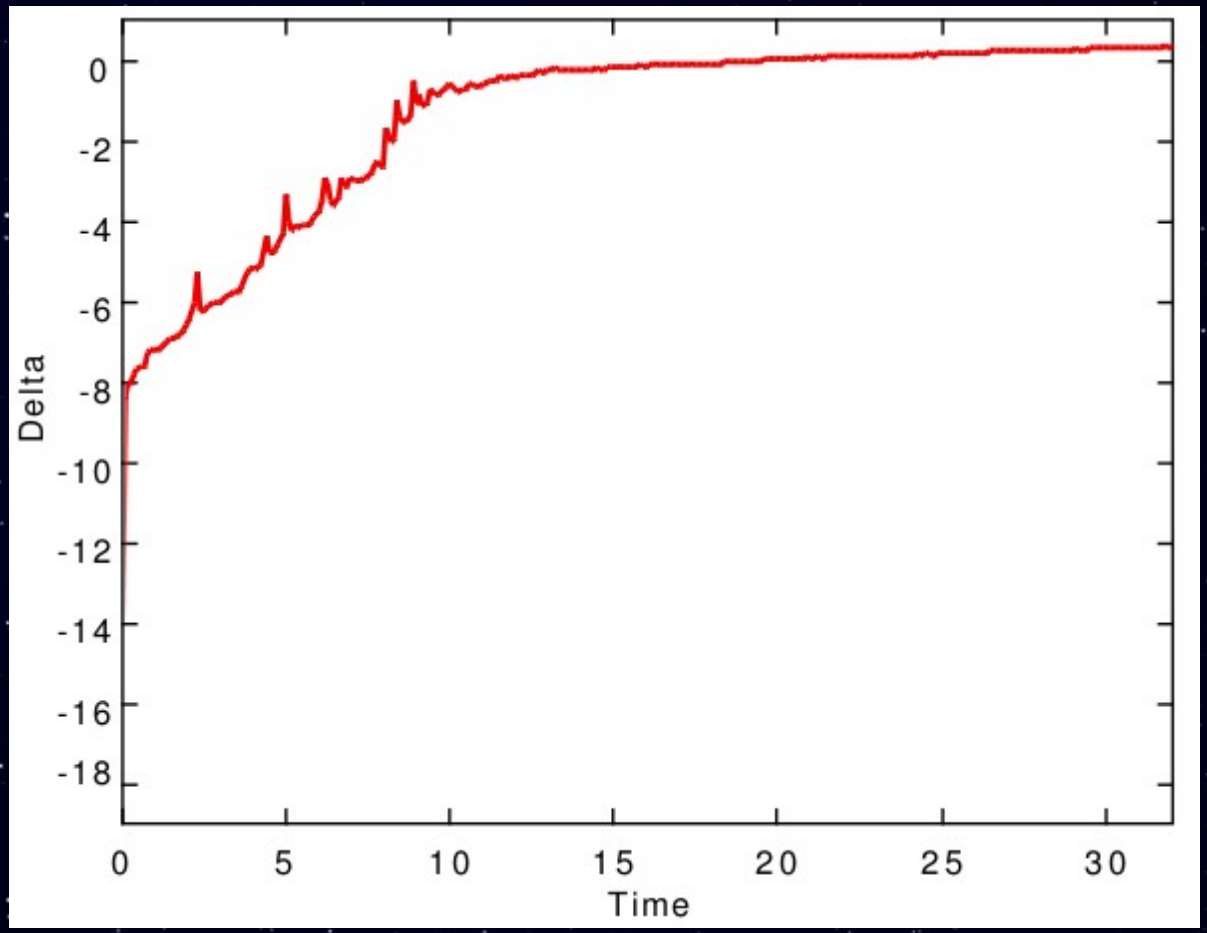

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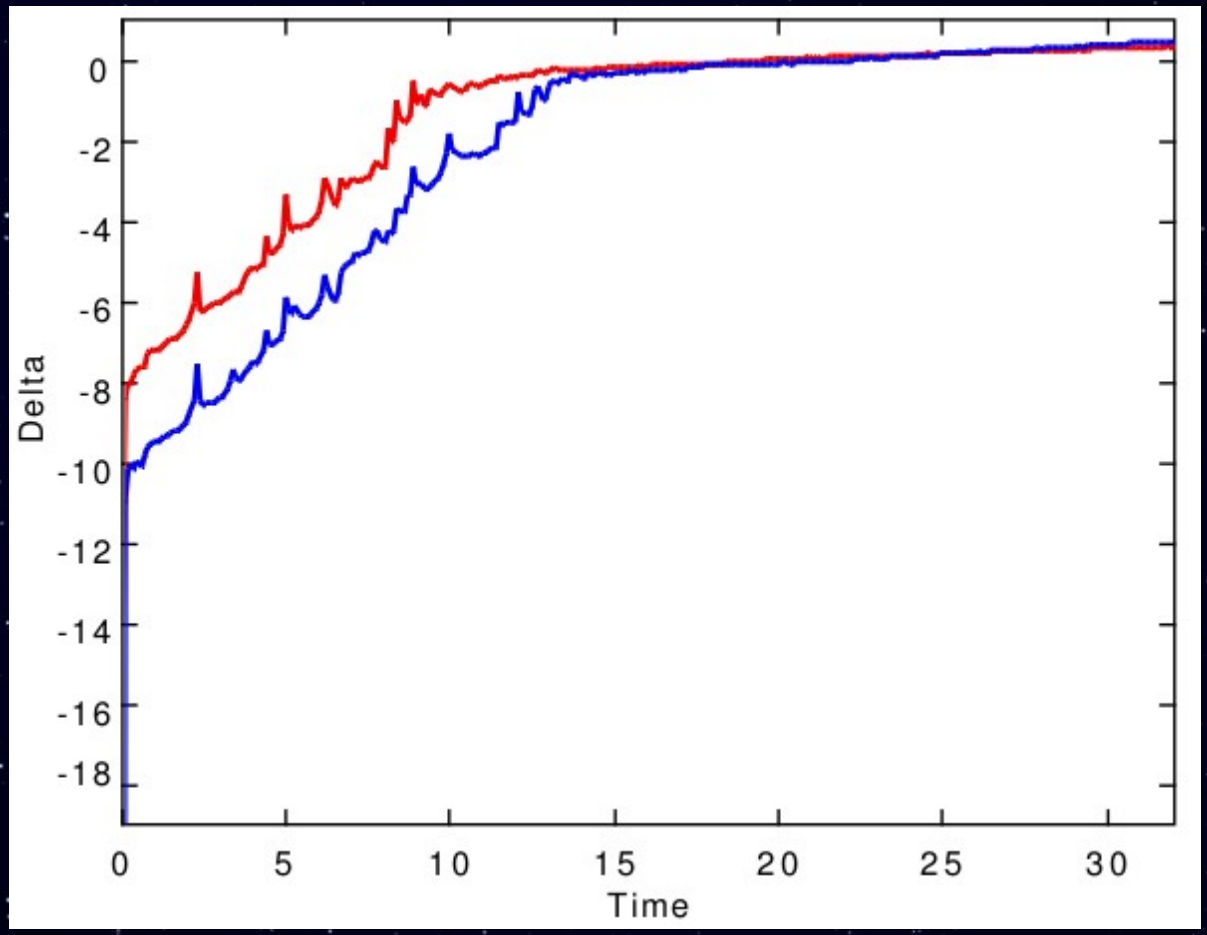

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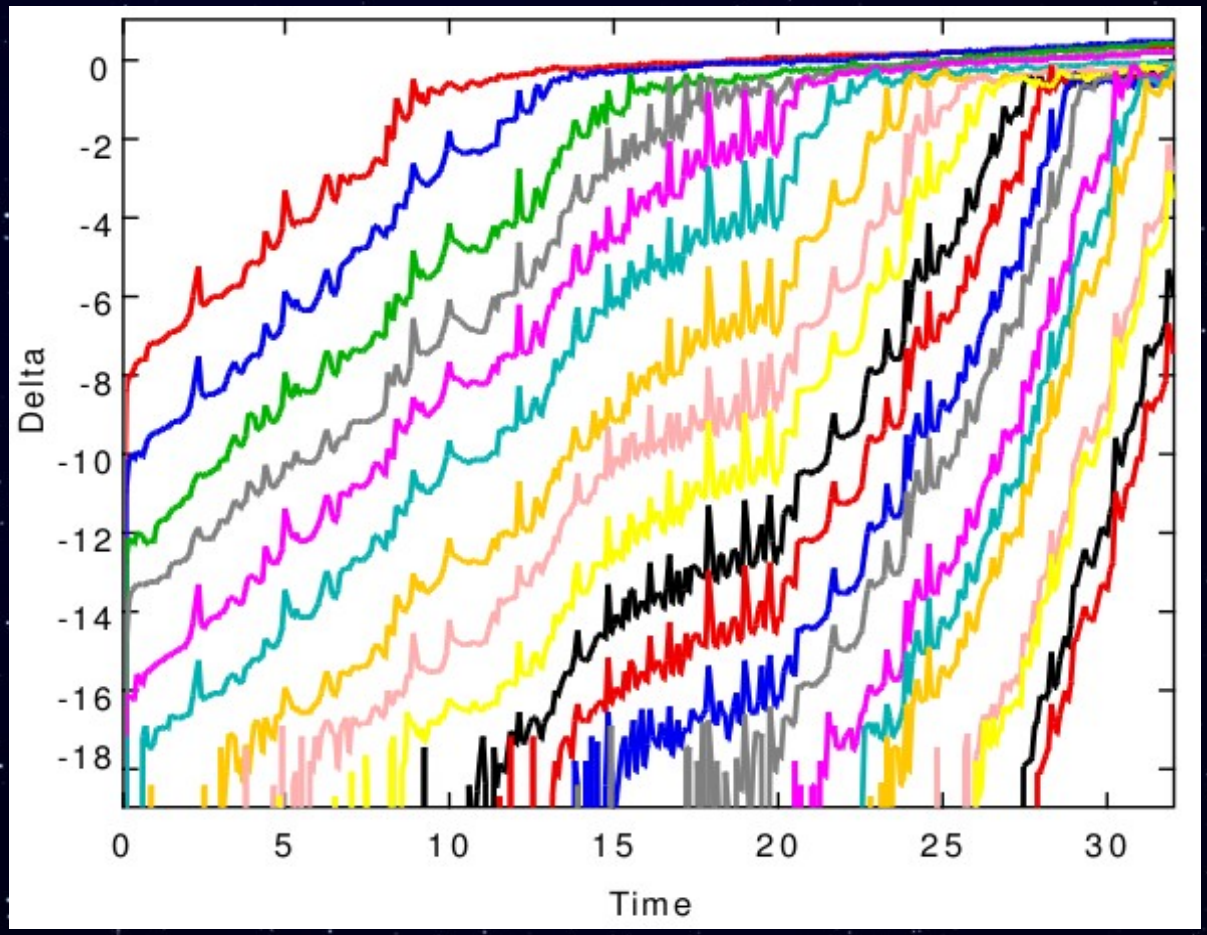

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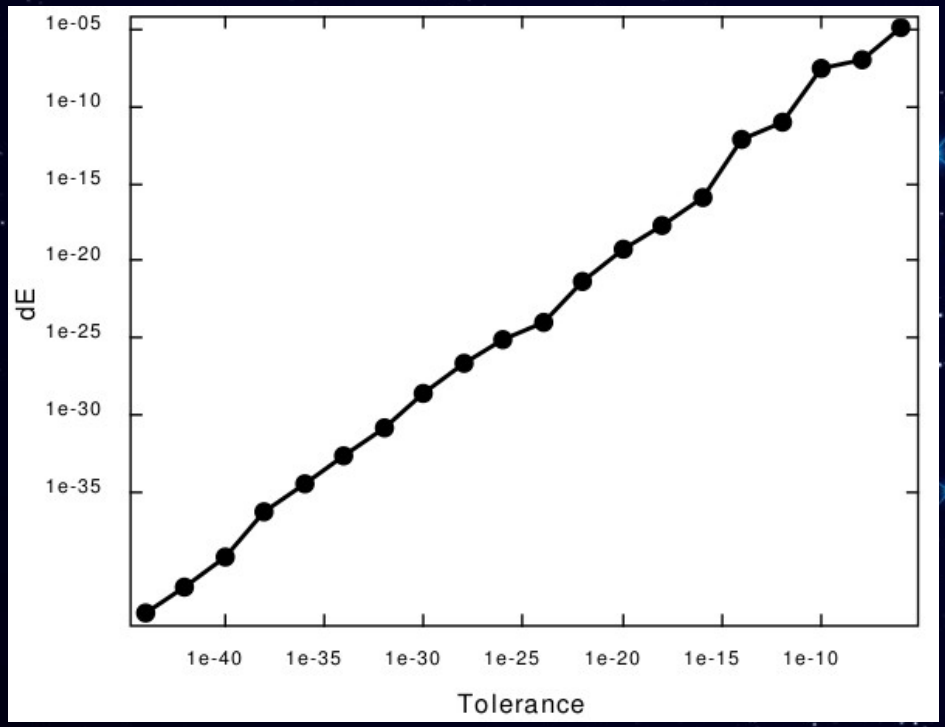
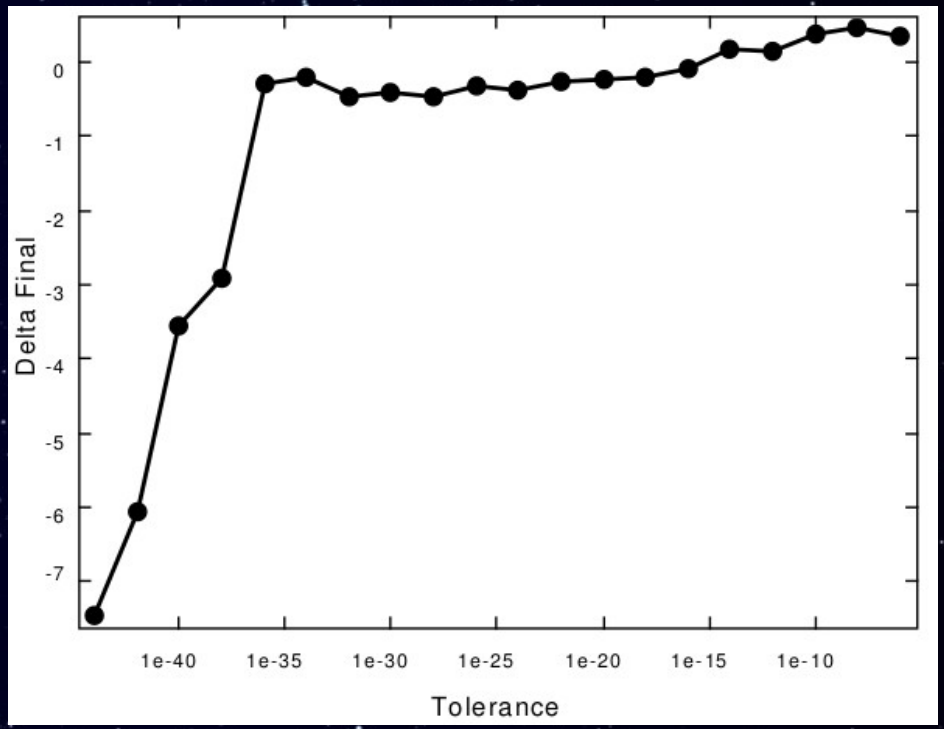




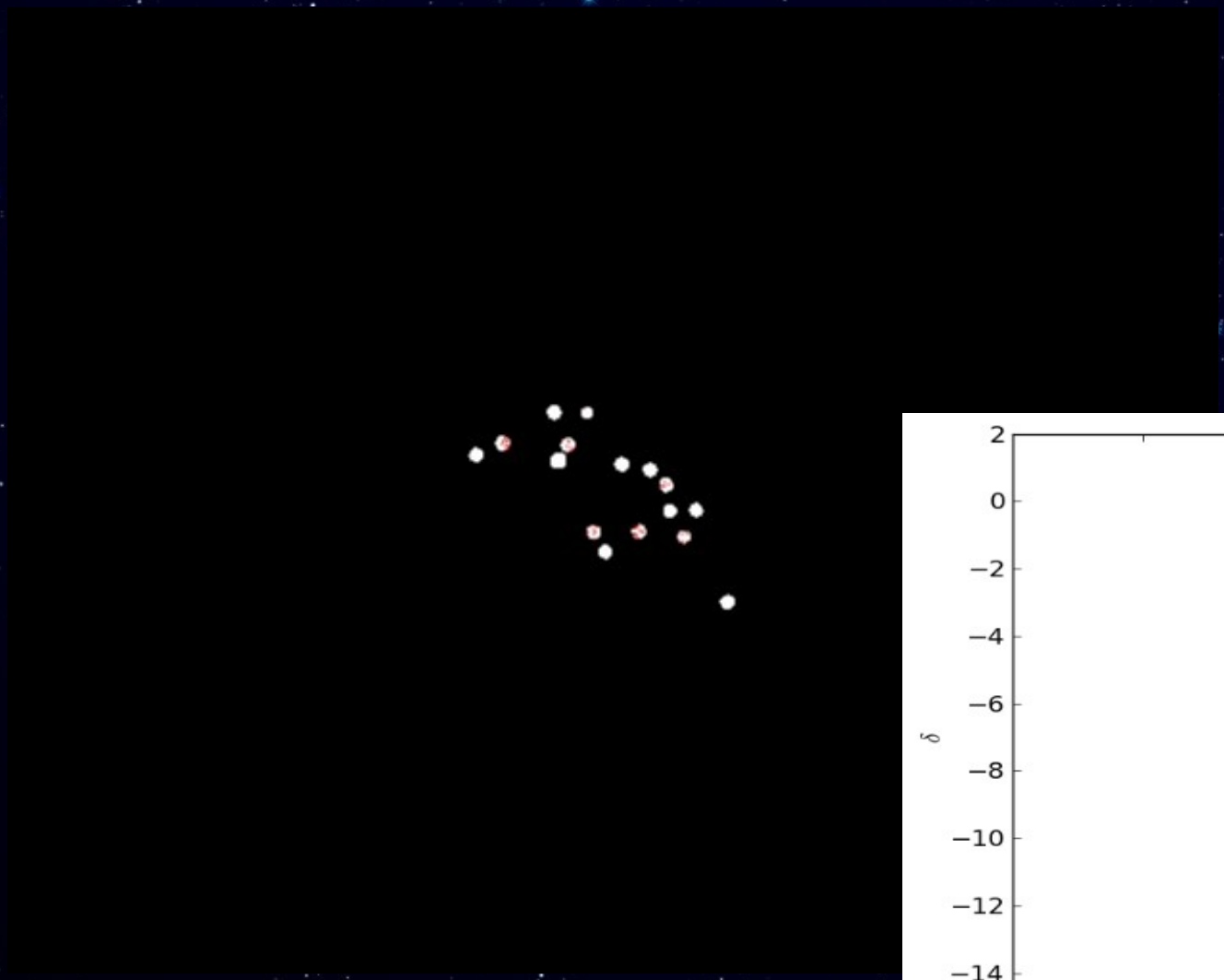
N=16 Plummer Experiment

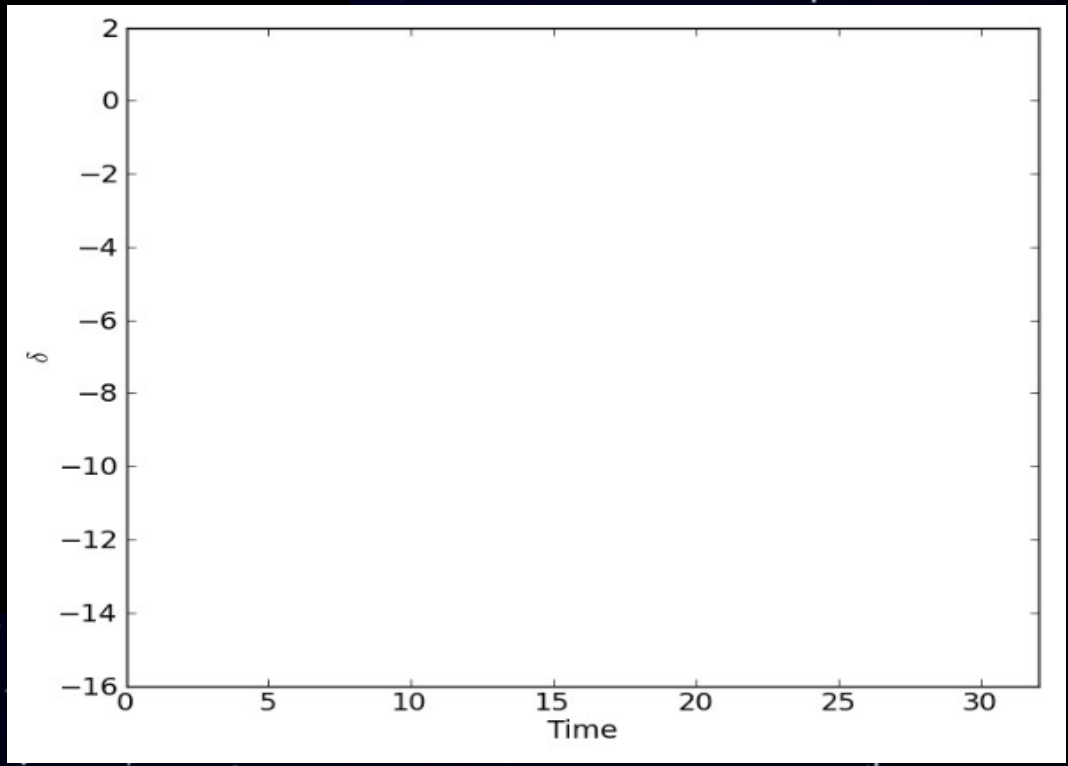
- N = 16, Tsim = 32

- Tol = 1e-42
- N_{bits} = 200

- Ncores = 4
- Tcpu = 2.5 hours
- dE = 5e-42

N=16 Plummer Experiment





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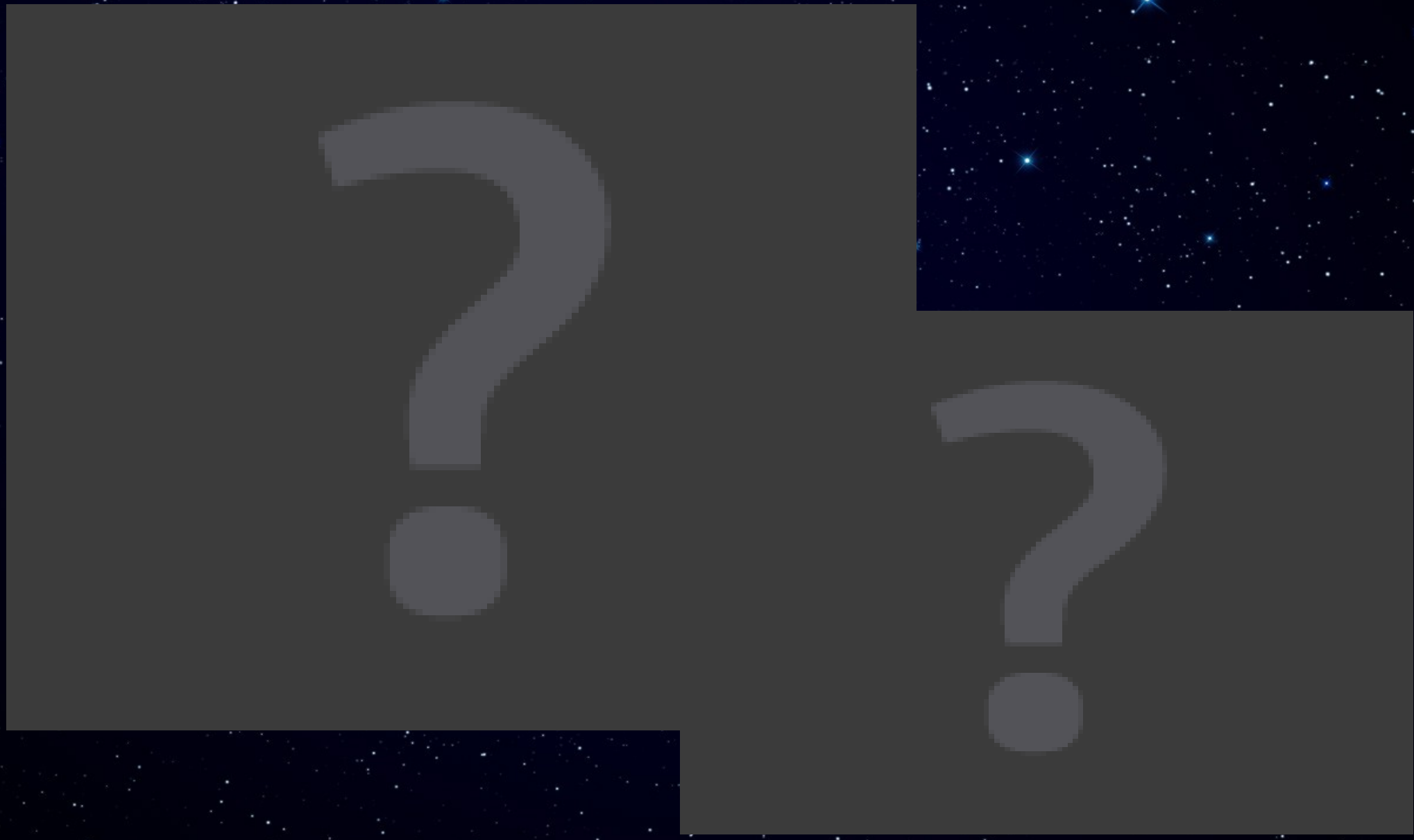

Part 2 – Statistical Accuracy

"*What degree of accuracy is enough?*"

(Smith 1979)

"*How badly are we allowed to integrate?*"

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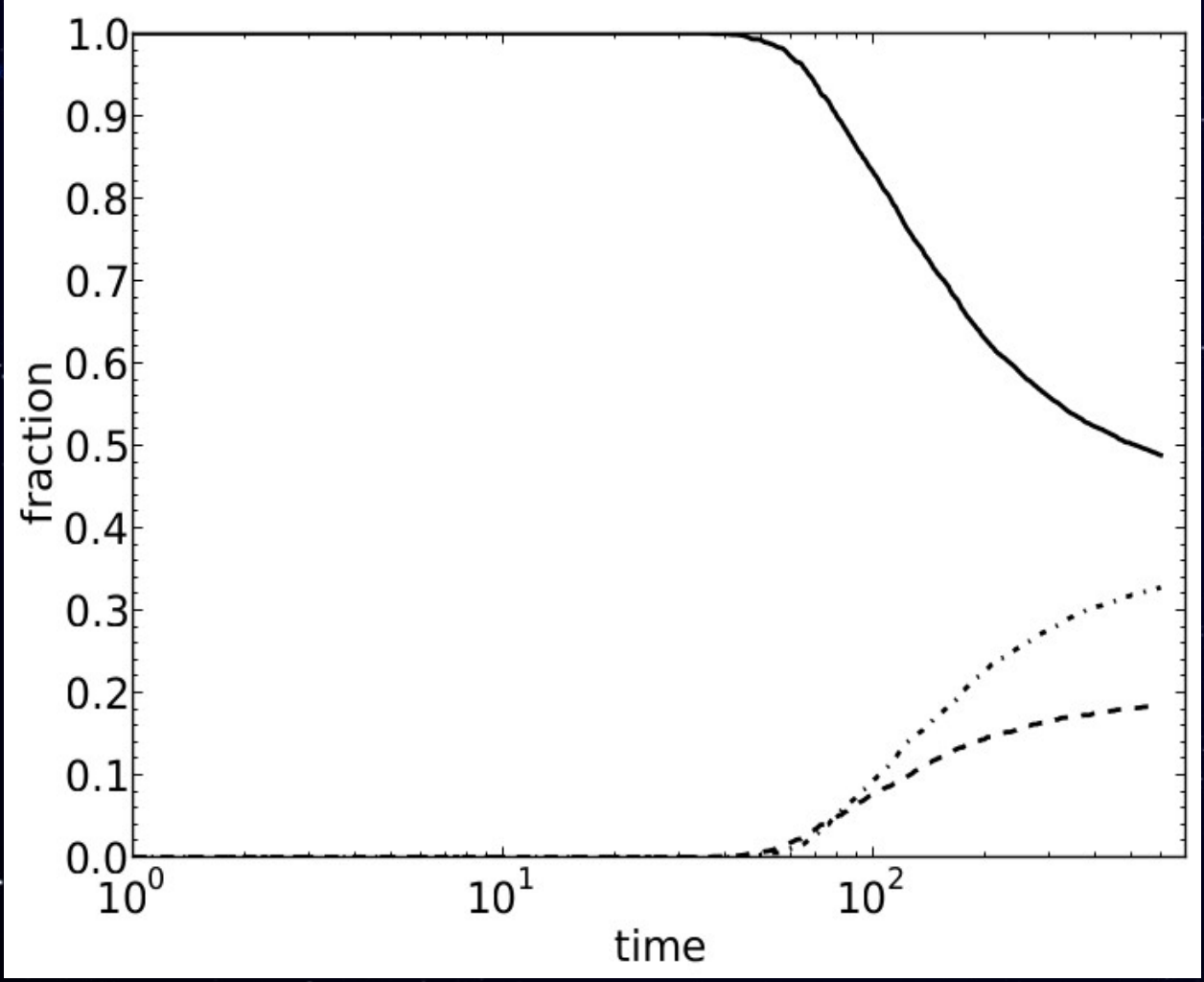
N=3 Statistics

- Initial Conditions : 10k random Plummers
- Stopping Condition : Binary + Escaper
- Integrators : LF, H4, BRUTUS
- Measure : Binary properties

N=3 Statistics

Hermite 4th order



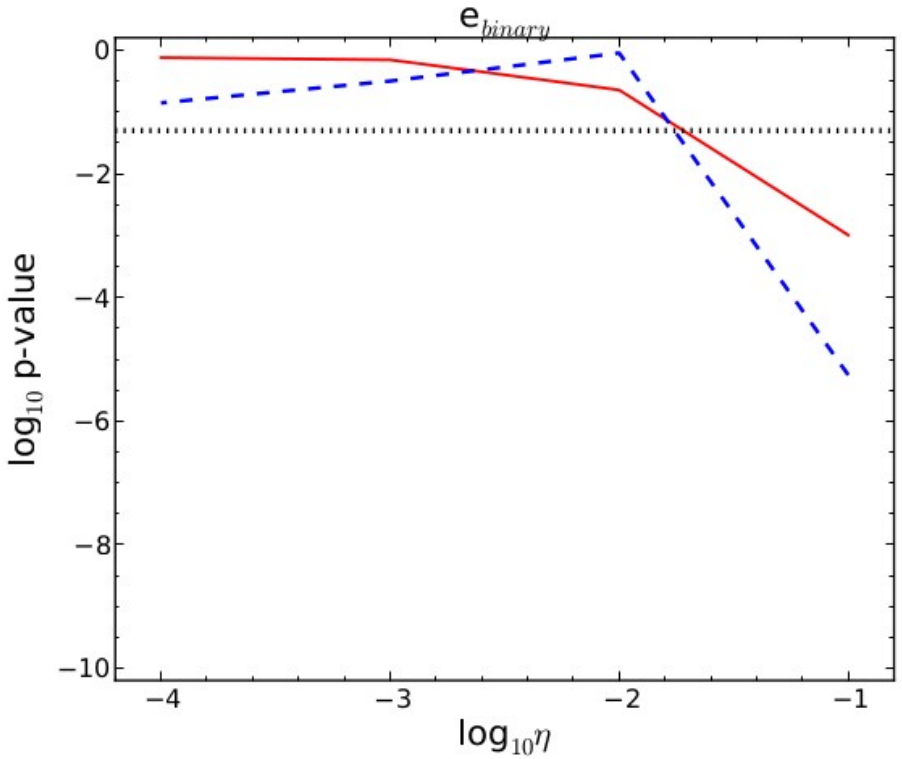
Categories:

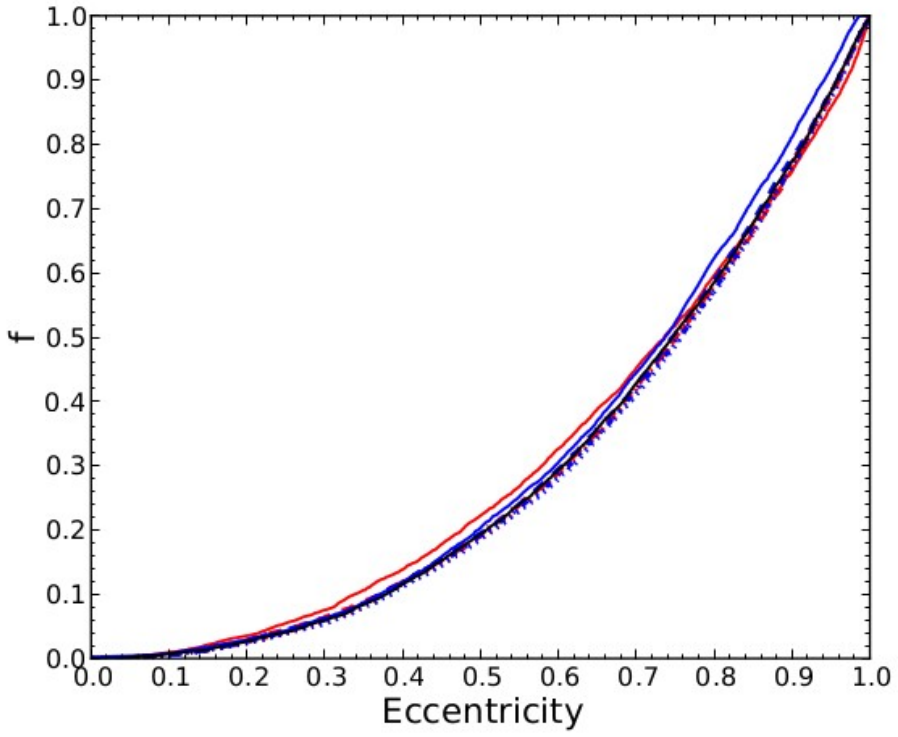
– = Accurate

-- = Similar

-. = Mixed

N=3 Statistics





Conclusions

"What degree of accuracy is enough?"

"How badly are we allowed to integrate?"

- Converged solutions can be obtained using BRUTUS
- The accuracy of conventional simulations can be determined by a comparison with BRUTUS
- Future studies will compare statistics of conventional and converged solutions in a direct way

Plans for the future

- Extend the BRUTUS simulations to higher N
- To provide a converged solution for a 1024-body system through core collapse