Roche volume filling of star clusters in the Milky Way

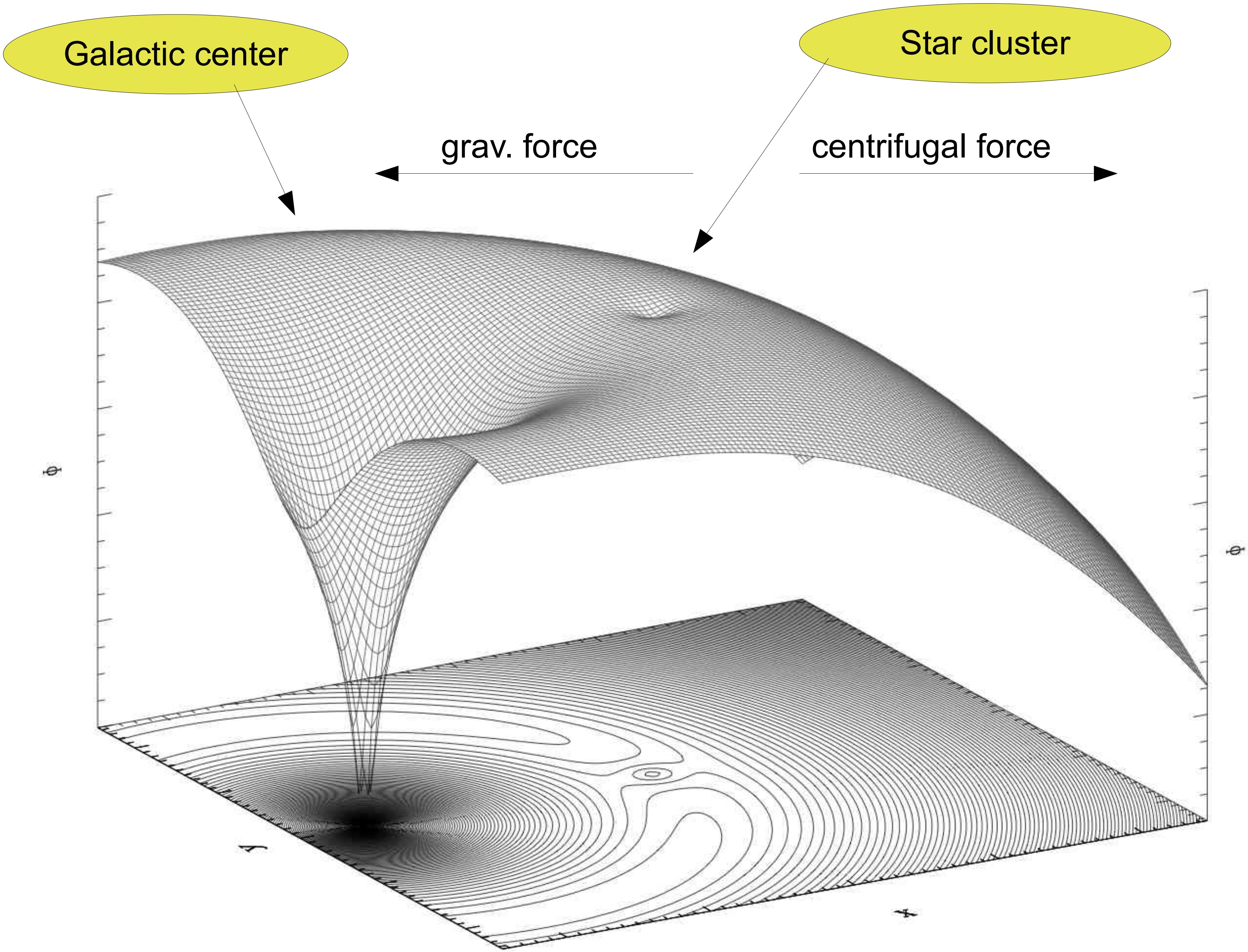
Andreas Ernst, ARI Heidelberg

Aarseth N-body Meeting 2012
Bonn, Dec. 3-5, 2012

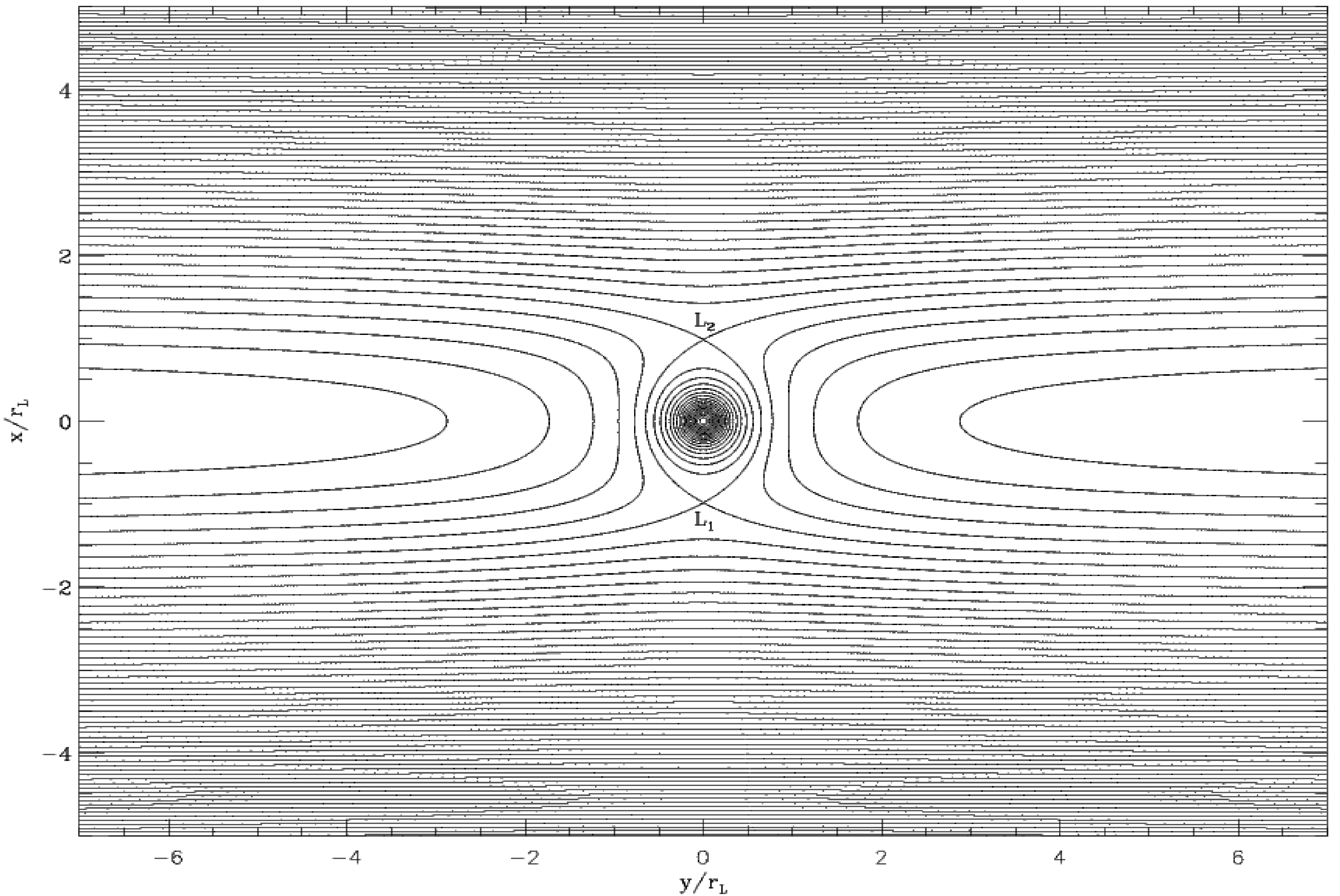
Outline

- Roche volume – what is it?
- Roche volume filling
- Resonances, Periodic orbits
- N-body simulation
- van den Bergh correlation
- Analysis of existing data sets
- Results

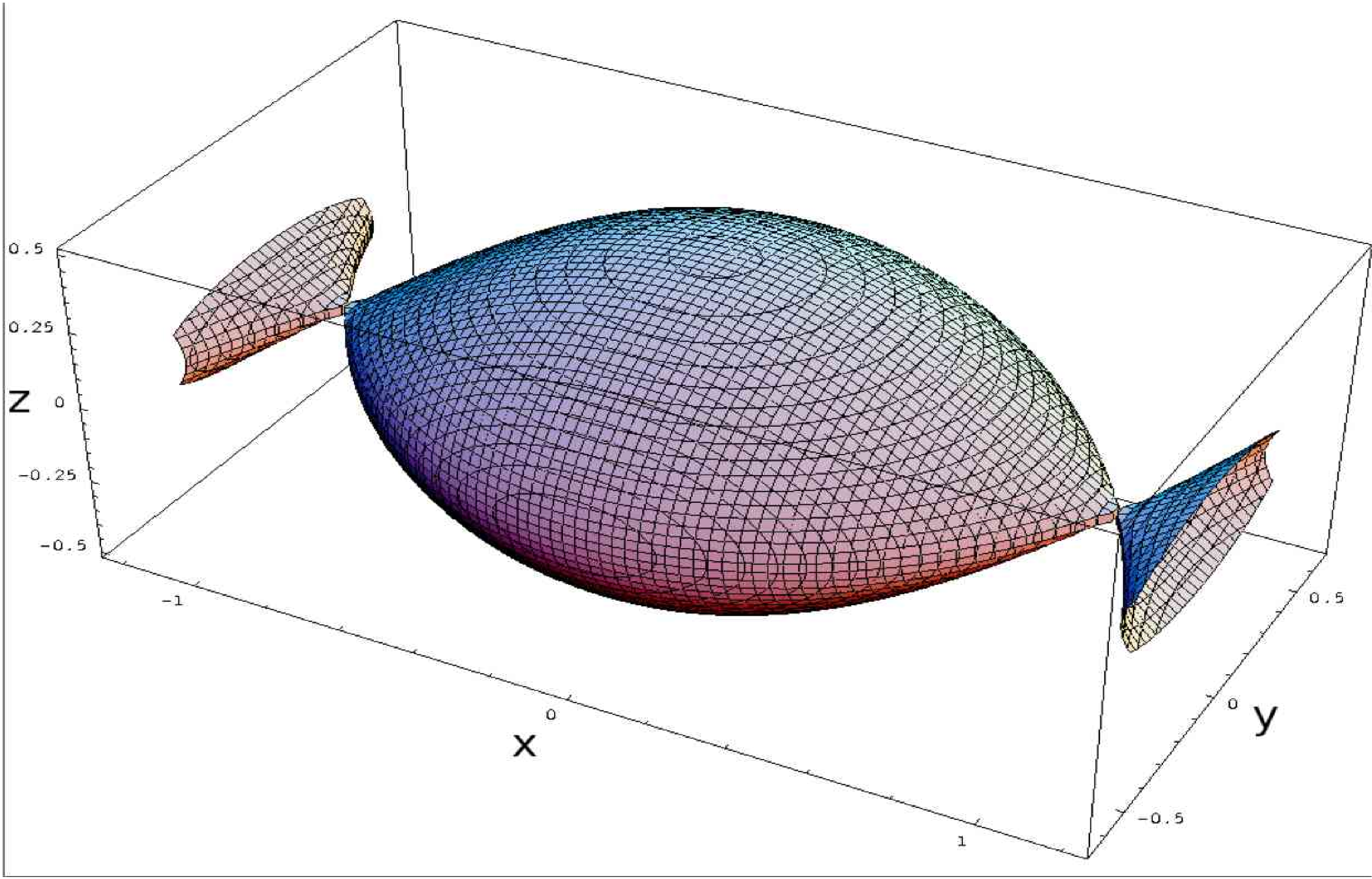
Star cluster in a tidal field



Jacobi radius



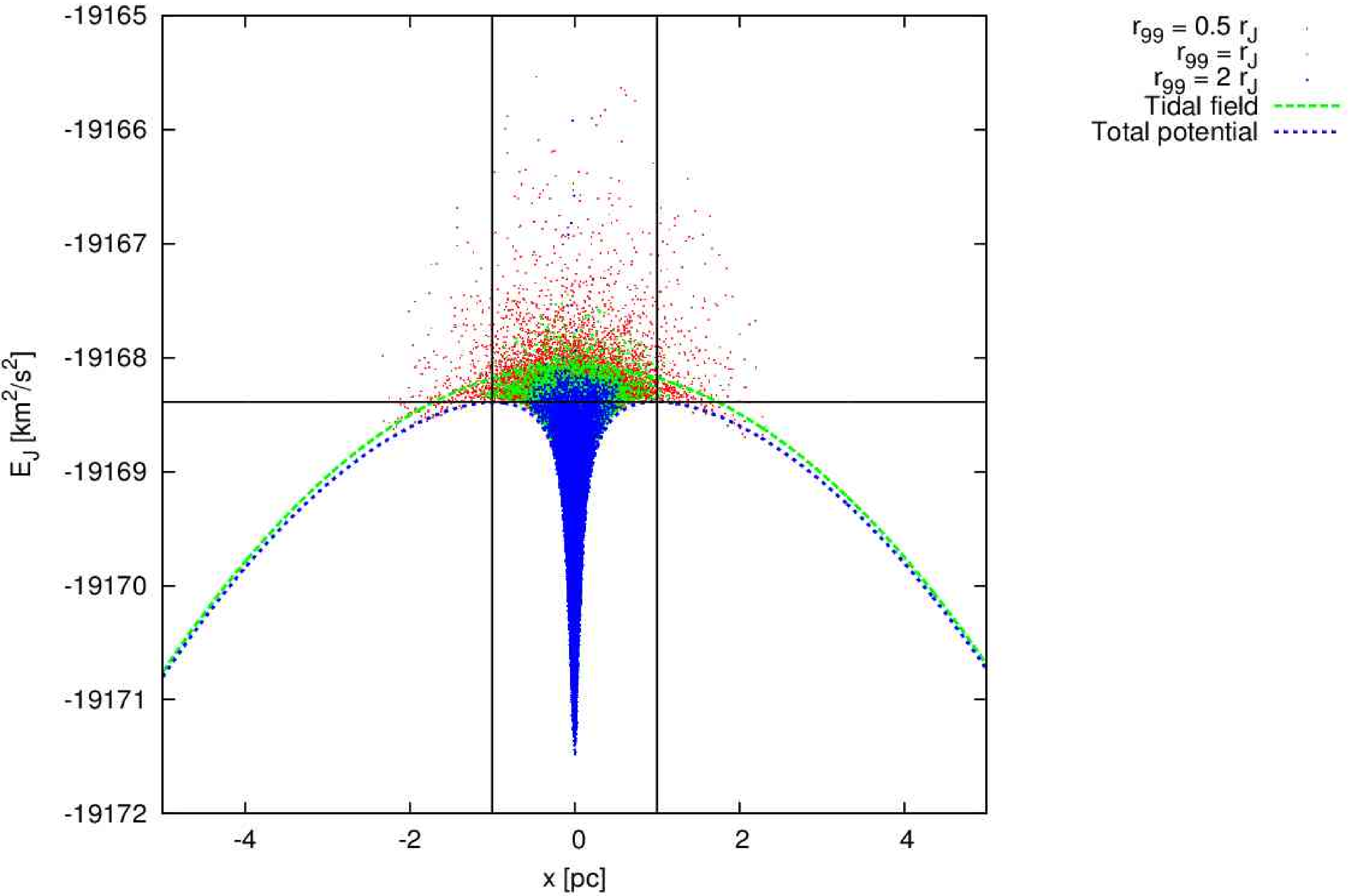
The Roche volume



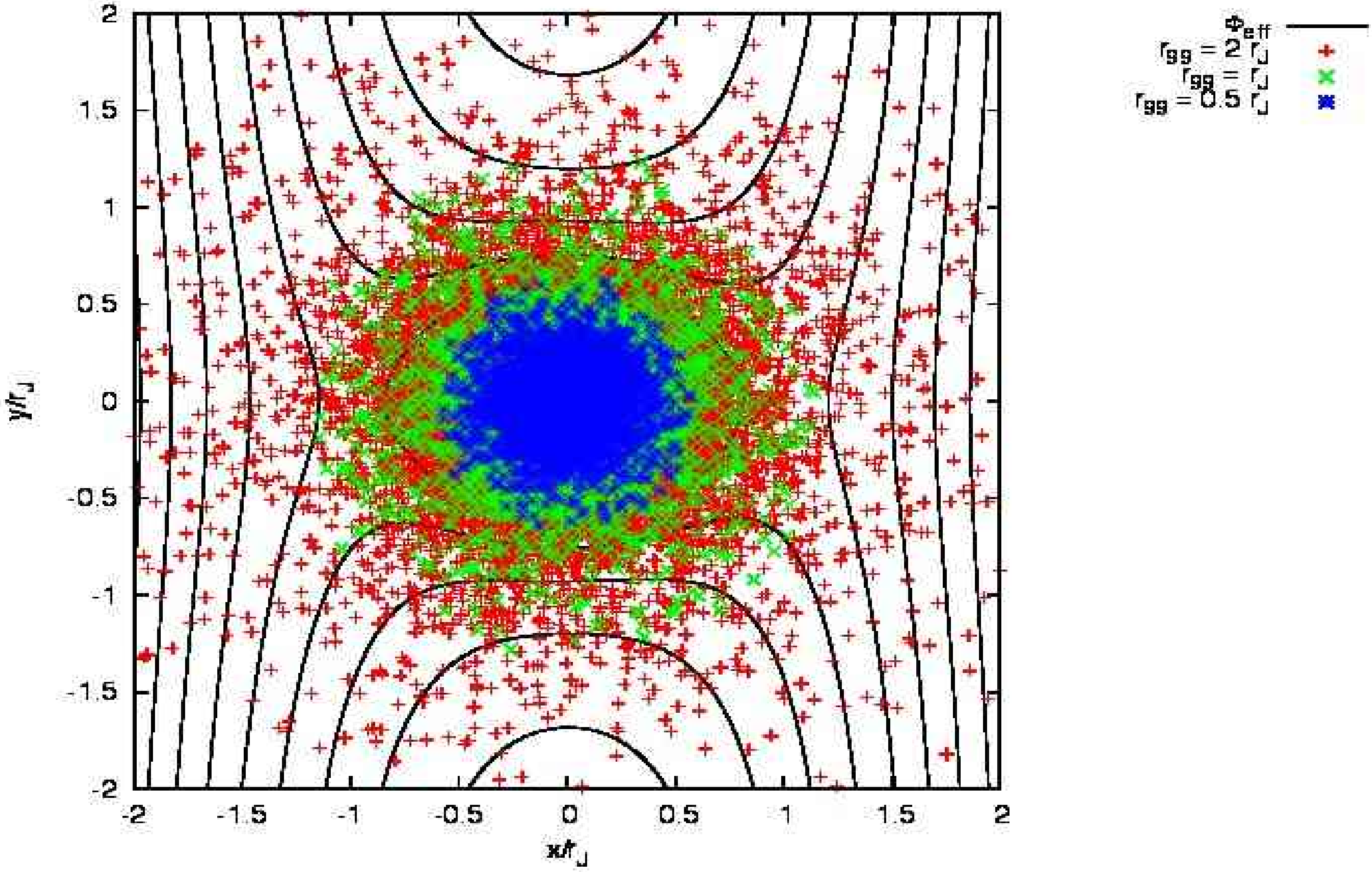
Questions / Thoughts

- To what extent do open and globular clusters fill the Roche volume?
- What is the difference between open and globular clusters with respect to Roche volume filling?
- Jacobi radius provides a natural scale
- Another scale is given by the position/width of the main resonant island

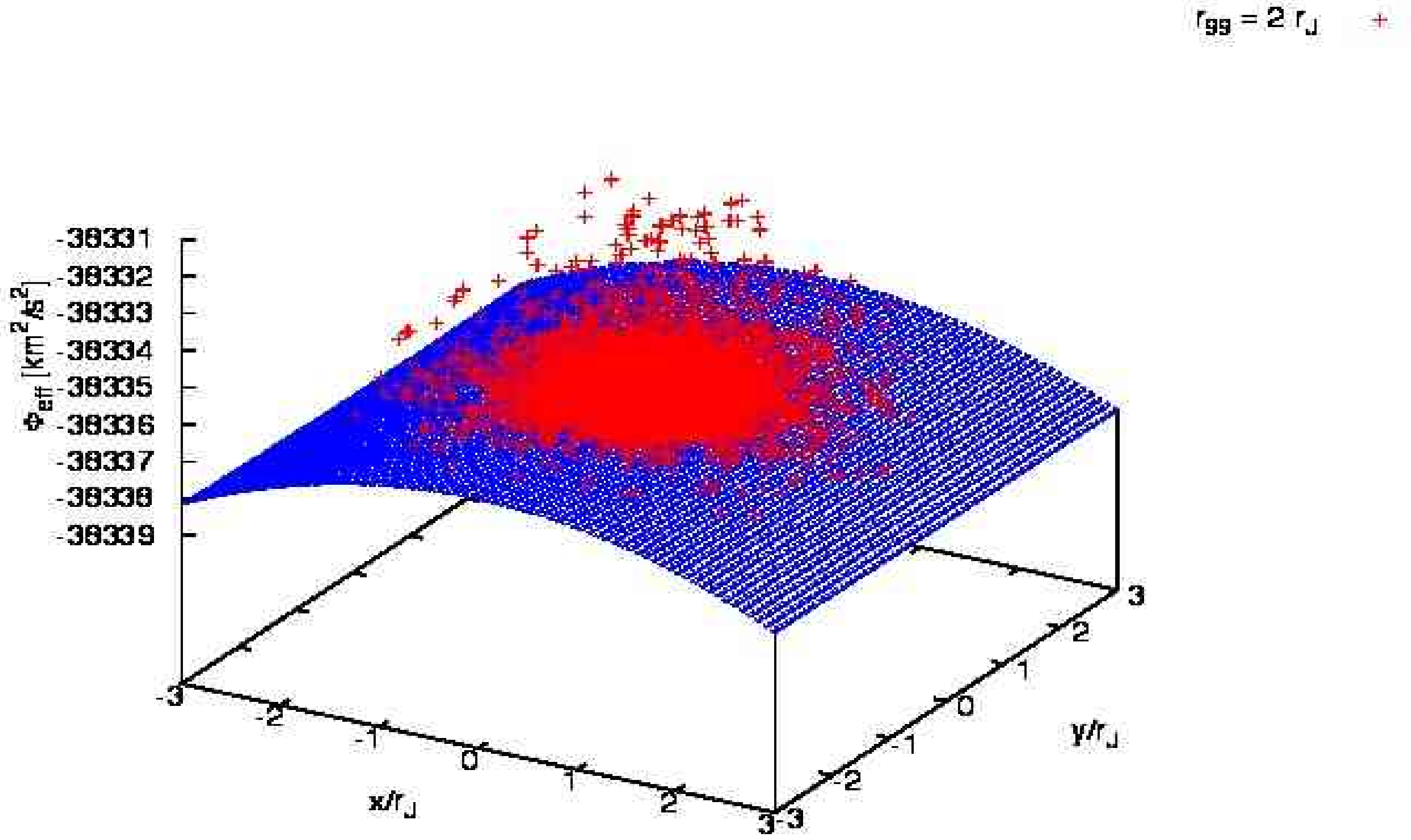
Roche-volume filling 1



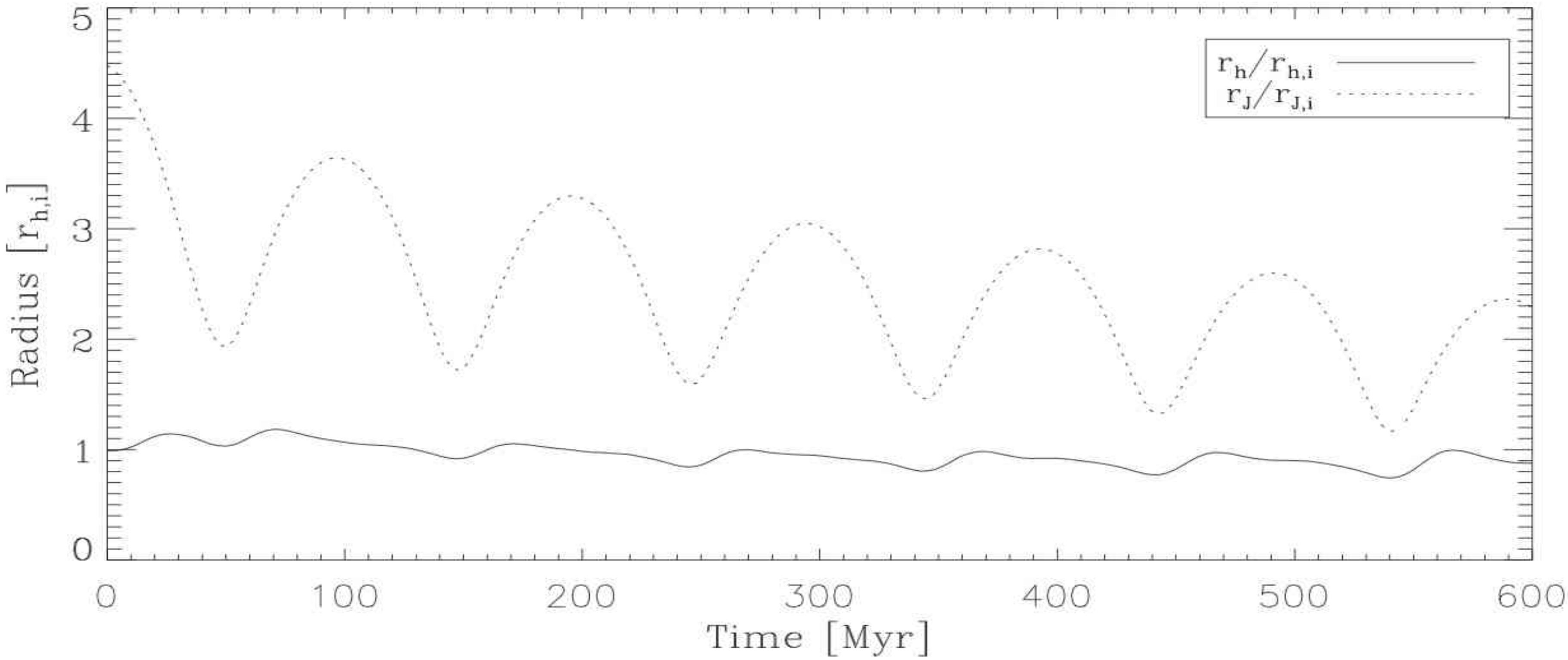
Roche volume filling 2



Roche volume filling 3



N-body simulation



- N=50.000
- Standard double segment Kroupa IMF
- Initially Roche volume filling

Van den Bergh (1994)

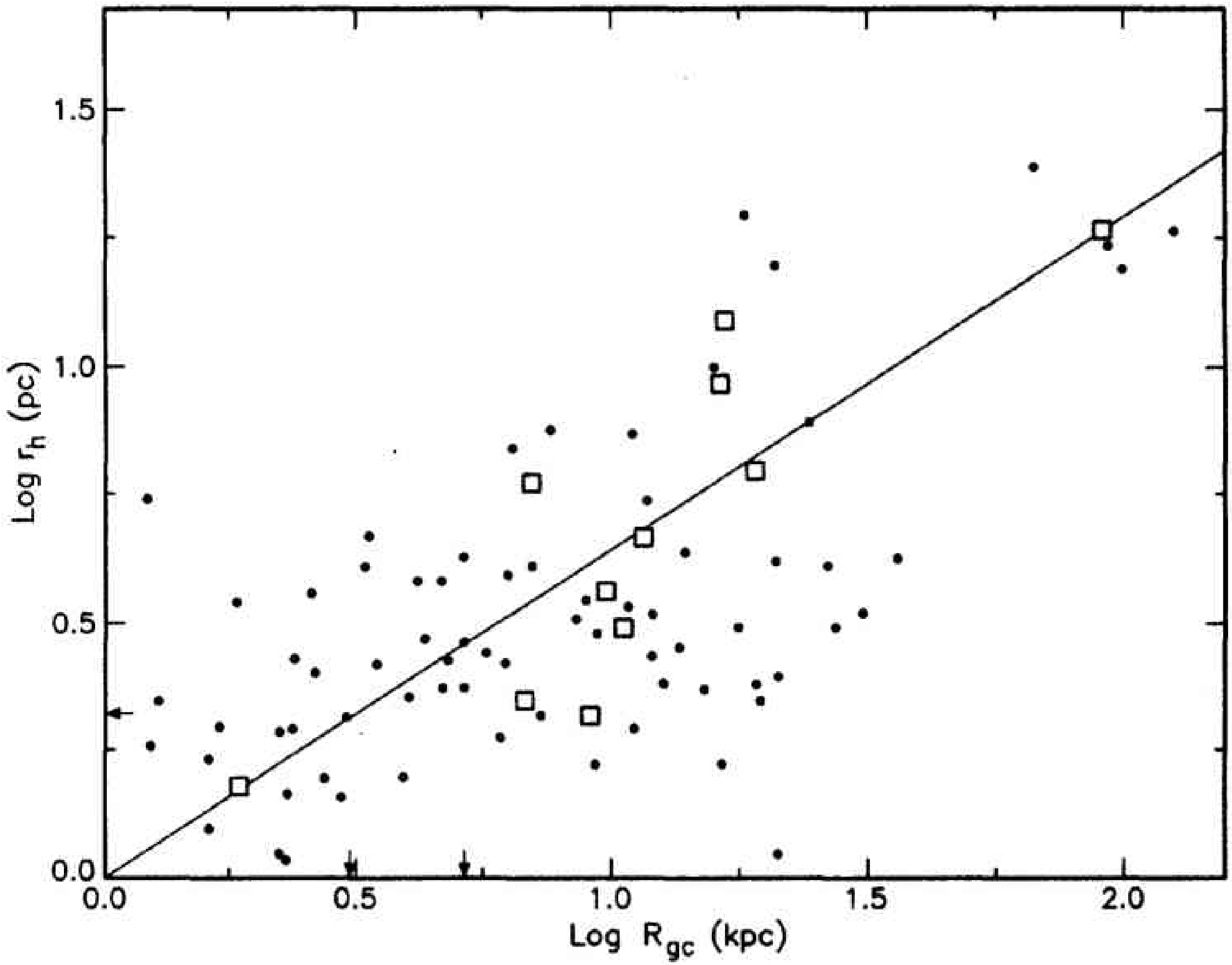


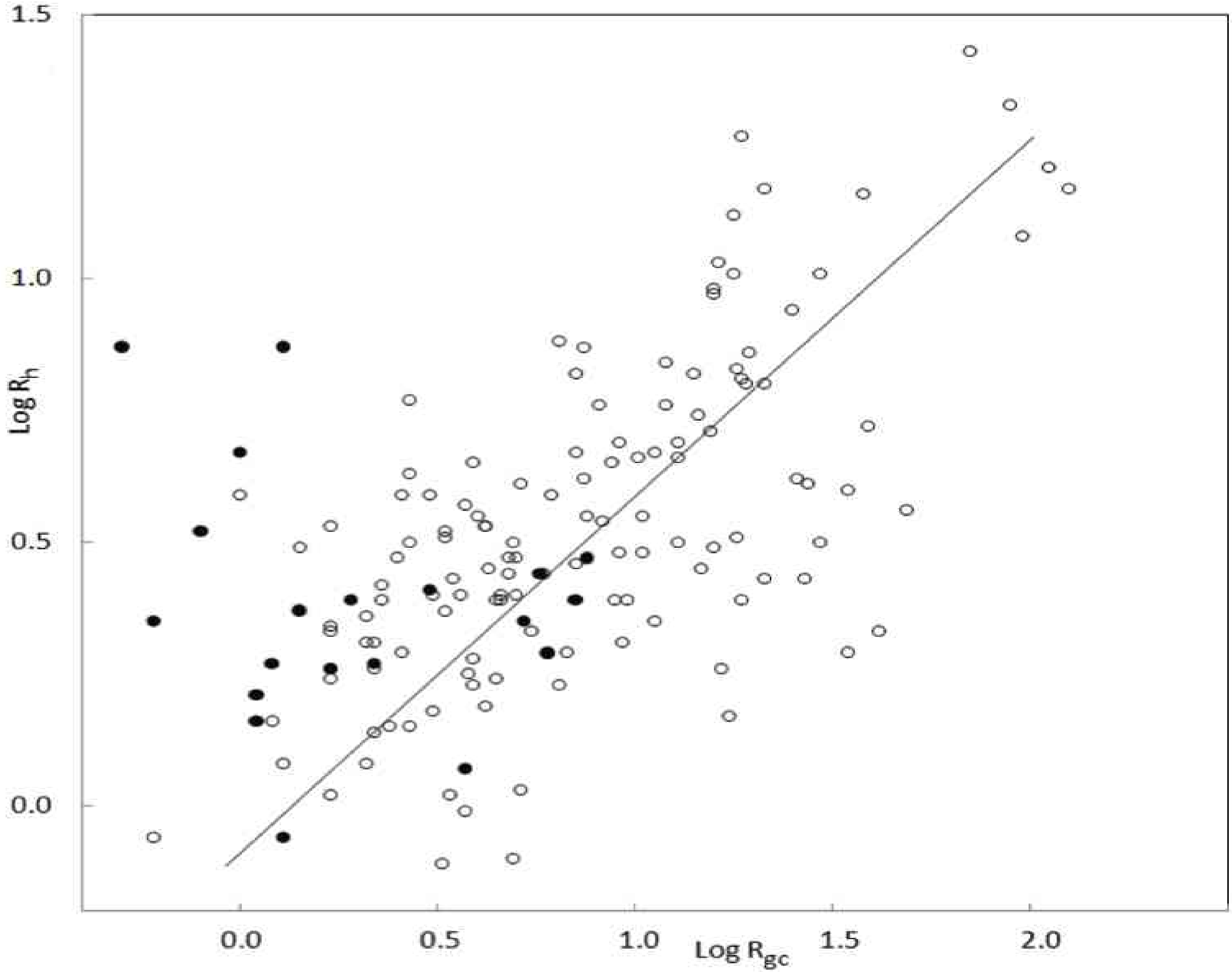
FIG. 1. Cluster radius vs Galactocentric distance for globular clusters with $-2.0<[Fe/H]<-1.0$ (dots) and $[Fe/H]<-2.0$ (squares). The fiducial line is the relation $\log r_h(pc)=0.65 \log R_{gc}(kpc)$. Clusters in both metallicity ranges appear to scatter about the same r_h vs R_{gc} relation.

Slope ≃ 2/3

Correlation coefficients:

0.85 [Fe/H]<-2.0
0.60 -2.0<[Fe/H]<-1.0
0.42 [Fe/H]>-1.0

Van den Bergh (2011)



Van den Bergh correlation

Jacobi radius (King, 1962):

$$r_J^3 = \frac{GM_{cl}}{\Omega^2 - \dfrac{d^2\Phi}{dR^2}}$$

Distance of the cluster center to the Lagrange points L_1 and L_2

Ω, Φ: angular velocity, gravitational potential of the galaxy

Van den Bergh correlation

Isothermal sphere:

$$\Omega^2 = \frac{L^2}{R^4}$$

$$L = ?$$

$$\Phi = V_C^2 \ln\left(\frac{R}{R_0}\right)$$

$$\frac{d\,\Phi}{dR} = \frac{V_C^2}{R}$$

$$\frac{d^2\Phi}{dR^2} = -\frac{V_C^2}{R^2}$$

Van den Bergh correlation

Isothermal sphere:

$$L = V_C R_P R_A \sqrt{\frac{2\ln\left(R_A / R_P\right)}{R_A^2 - R_P^2}}$$

Angular momentum of an orbit as a function of peri- and apocenter

Van den Bergh correlation

Jacobi radius

$$r_J^3 = \frac{GM_{cl}(R_A^2 - R_P^2)R^4}{2V_C^2 R_P^2 R_A^2 \ln(R_A/R_P) + V_C^2(R_A^2 - R_P^2)R^2}$$

Define guiding radius $R_g = \dfrac{L}{V_C}$

$$r_J = \left(\frac{GM_{cl}}{V_C^2}\right)^{1/3} \left(\frac{R^4}{R_g^2 + R^2}\right)^{1/3}$$

Van den Bergh correlation

Van den Bergh (1994, 2011)

$$r_h \propto R_{GC}^{2/3}$$

Jacobi radius in an isothermal halo:

$$r_J \propto R_{GC}^{2/3} \qquad \text{for } R_{GC} \gg L / V_C$$

These proportionalities and the assumption of an isothermal halo imply that

$$\frac{r_h}{r_J} \text{ is independent of } R_{GC}$$

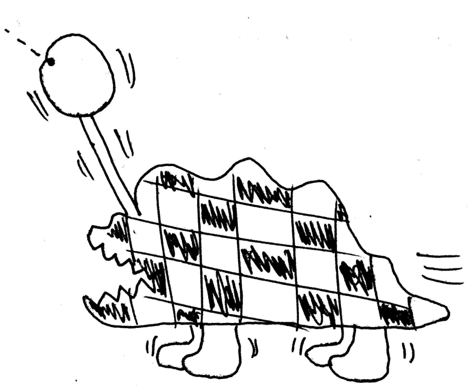
for Milky Way GCs within the scatter of the correlation.

The ratio r_h/r_J

$$r_J = \left[\frac{GM_{cl}}{(4-\beta^2)\Omega^2}\right]^{1/3}, \qquad \beta = \frac{\kappa}{\Omega}, \qquad M_{cl} = \frac{8\pi}{3}\rho r_h^3$$

for OCs:
$$\frac{r_h}{r_J} = \left[\frac{3(4-\beta^2)\Omega^2}{4\pi G\rho}\right]^{1/3} = \left(\frac{4-\beta^2}{2}\right)^{1/3}\left(\frac{t_{orb}}{T_{orb}}\right)^{2/3}$$

for GCs:
$$\frac{r_h}{r_J(t)} = \left(\frac{t_{orb}}{2\pi}\right)^{2/3}\left(\frac{V_C^2}{2}\right)^{1/3}\left(\frac{R_g^2 + R(t)^2}{R(t)^4}\right)^{2/3}$$



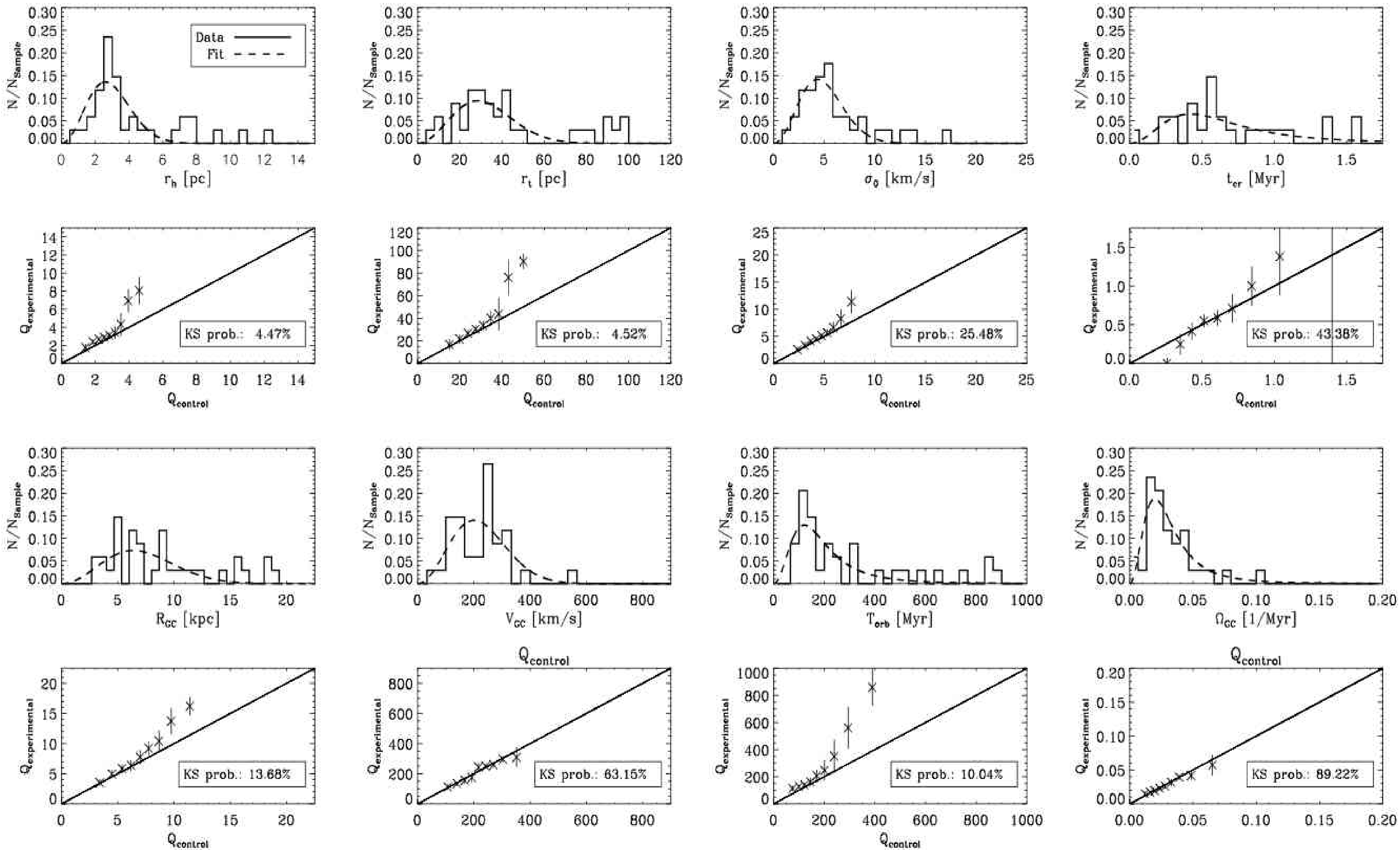
Values for GCs and OCs

OC Parameter	Value
Sample size N_{OCs}	236
Median half-mass radius r_h [pc]	1.94 ± 0.15
Median tidal radius r_t [pc]	7.90 ± 0.51
Velocity dispersion σ_0 [pc Myr^{-1}]	0.31
Median crossing time $t_{\mathrm{cr,OC}} = r_h/\sigma_0$ [Myr]	6.26 ± 0.48
Average orbital period $T_{\mathrm{orb,OC}}$ [Myr]	220 ± 30
Average eccentricity e_{OC}	0.127 ± 0.003

GC Parameter	Value
Sample size N_{GCs}	34 (38)
Median half-light radius r_h [pc]	3.13 ± 0.51
Median tidal radius r_t [pc]	33.02 ± 4.83
Median velocity disp. σ_0 [pc Myr^{-1}]	5.11 ± 0.64
Median crossing time $t_{\mathrm{cr,GC}} = r_h/\sigma_0$ [Myr]	0.587 ± 0.363
Median Galactocentric radius $R_{\mathrm{orb,GC}}$ [kpc]	7.75 ± 0.84
Median velocity V_{GC} [pc Myr^{-1}]	249 ± 18
Median orbital period $T_{\mathrm{orb,GC}}$ [Myr]	207 ± 54
Median angular speed Ω_{GCs} [Myr^{-1}]	0.0259 ± 0.0036
Median eccentricity e_{GC}	0.622 ± 0.044

Samples by Piskunov et al. (2007), Dinescu et al. (1999), Harris (1996, 2010 edition)

Distr. of GC parameters



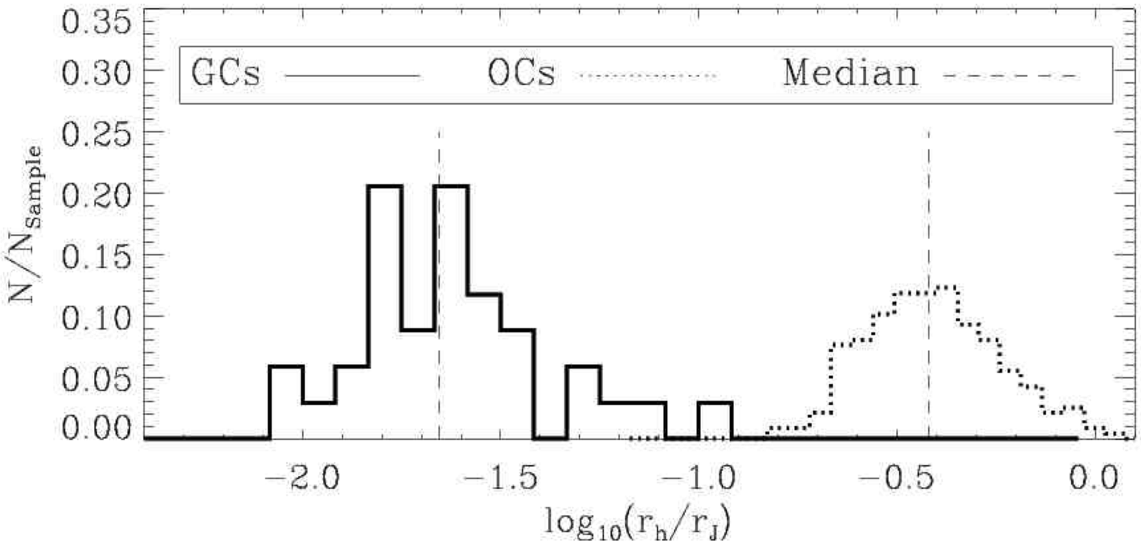
Fitting results for GCs

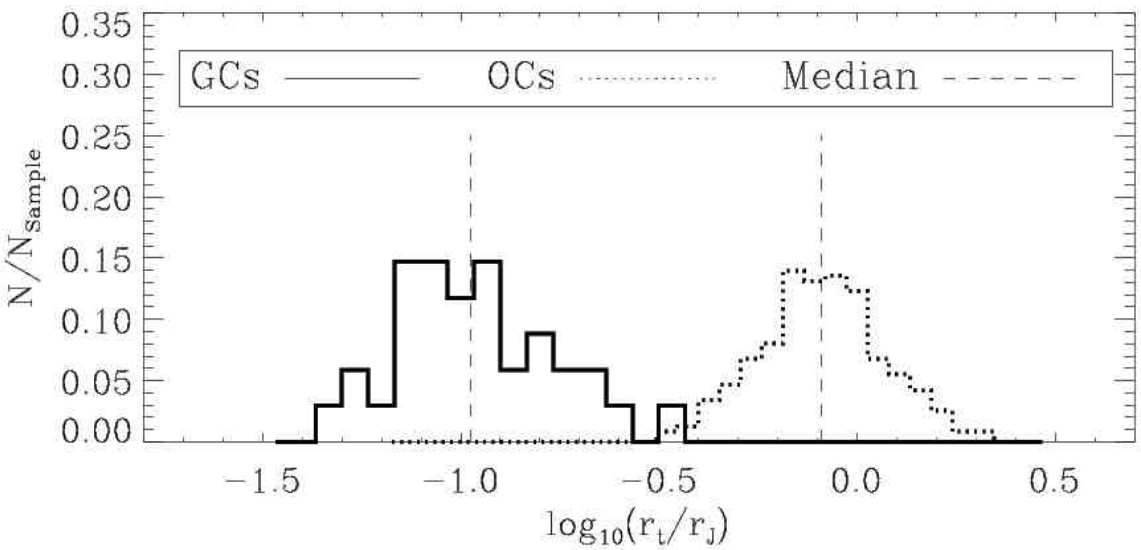
Table 2. Results of the fitting with analytical distribution functions and Kolmogorov-Smirnov (KS) tests. The given parameters are those of the fits/control samples.

Parameter	σ_{1D}	Mean	Median	$P_{\rm KS}$ [%]
$r_{\rm h,GCs}$ [pc]	1.82	2.90	2.83	4.47
$r_{\rm t,GCs}$ [pc]	19.2	30.6	31.05	4.52
$\sigma_{0,\rm GCs}$ [km s^{-1}]	3.07	4.90	4.66	25.48
$R_{\rm GCs}$ [kpc]	4.39	7.01	6.98	13.68
$V_{\rm GCs}$ [km s^{-1}]	146	233	216	63.15

Parameter	σ_2/σ_1	Mean	Median	$P_{\rm KS}$ [%]
$t_{\rm cr,GCs}$ [Myr]	0.659	0.839	0.659	43.38
$T_{\rm orb,GCs}$ [Myr]	171	218	171	10.04
$\Omega_{\rm GCs}$ [Myr^{-1}]	0.0203	0.0258	0.0203	89.22

r_h/r_J and r_t/r_J for GCs and OCs





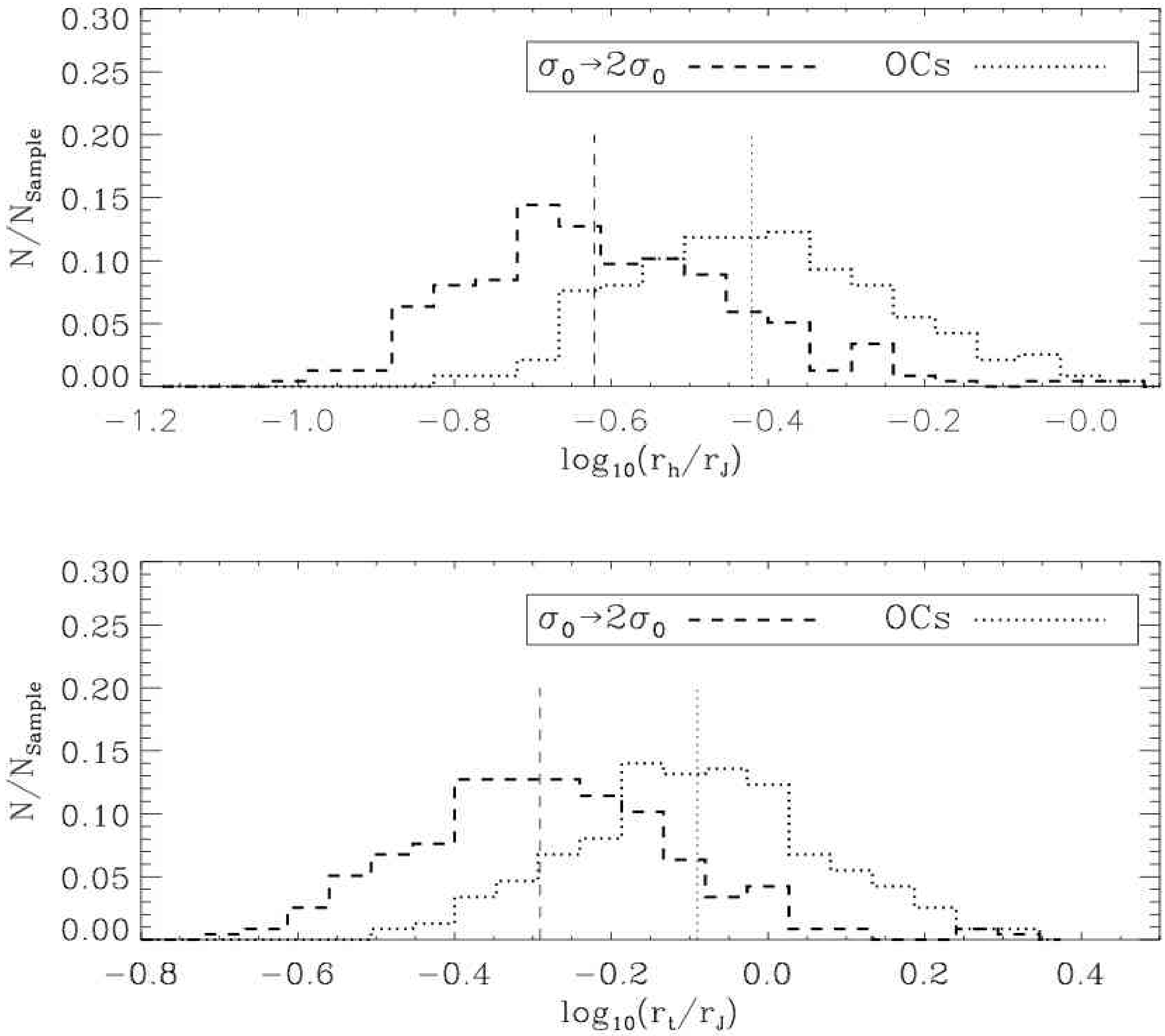
- Two distinct populations with respect to r_h/r_J

- r_h/r_J of OCs is larger

- OCs: King W_0=2-3

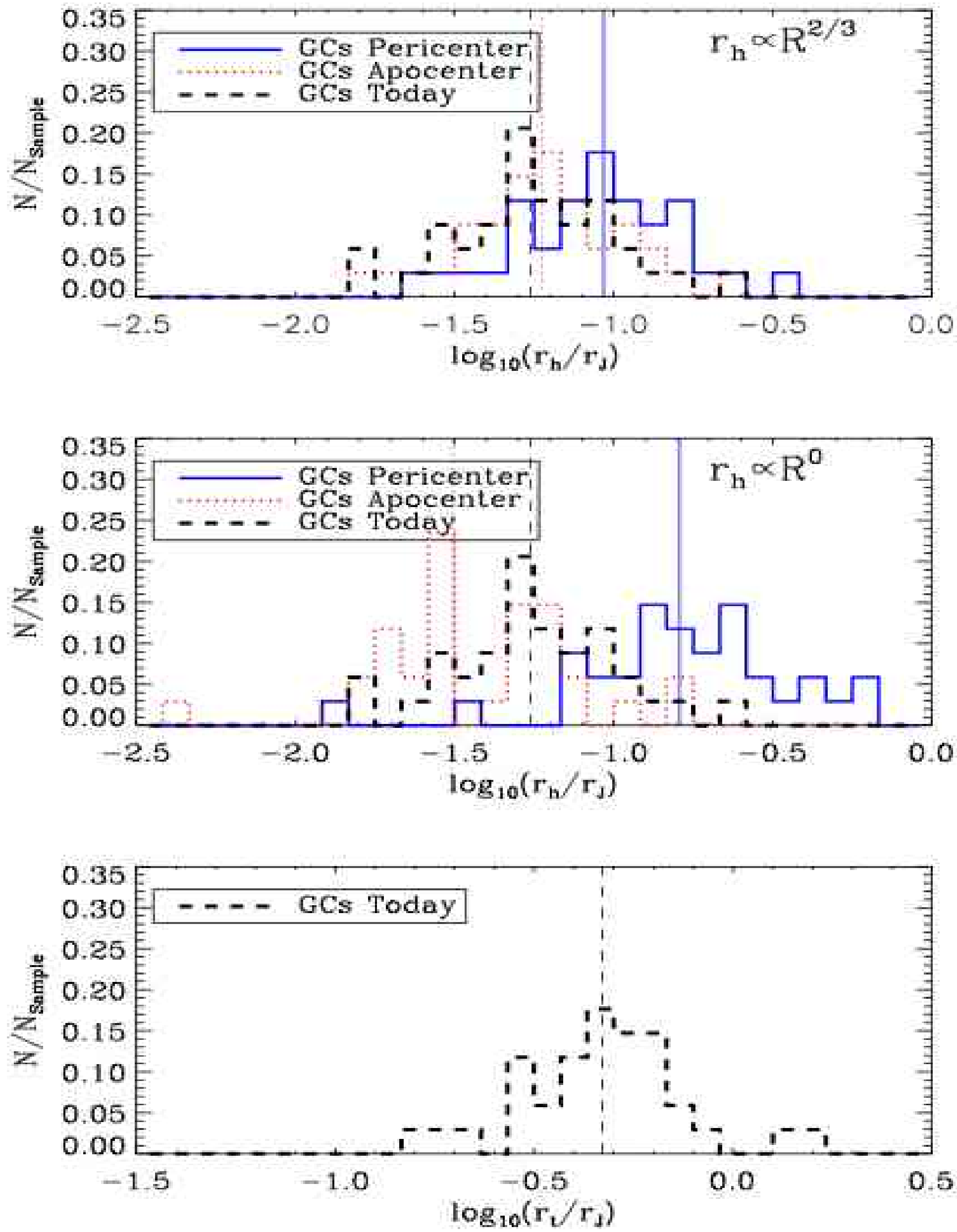
- GCs: King W_0=5-6

r_h/r_J and r_t/r_J for GCs and OCs



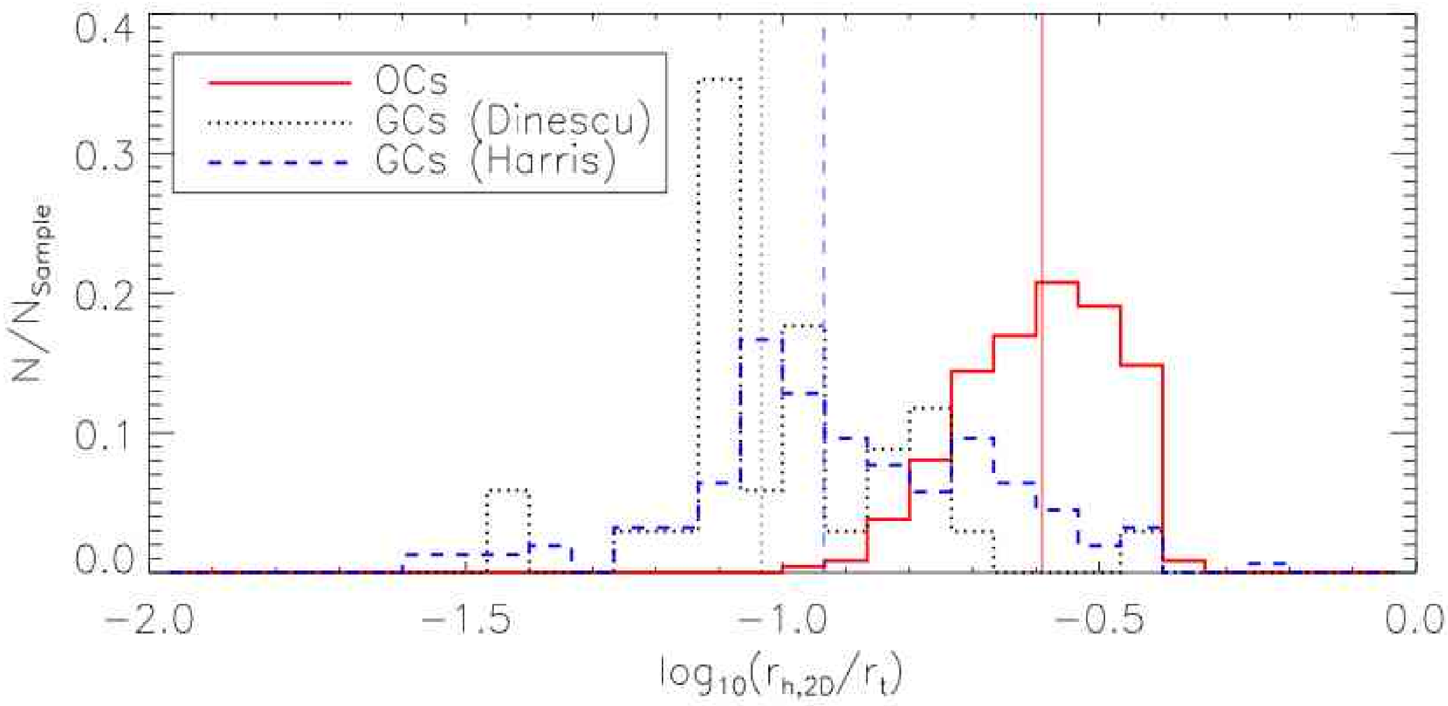
Ernst & Just, submitted (2012)

r_h/r_J and r_t/r_J for GCs and OCs



Ernst & Just, submitted (2012)

r_h/r_J and r_t/r_J for GCs and OCs

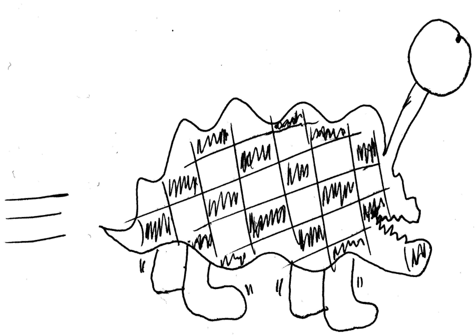


$r_{h,2D}$ = projected half-mass radius (OCs) or half-light radius (GCs)

Ernst & Just, submitted (2012)

Conclusions

- GCs are presently Roche volume underfilling
- In the pericenters of their orbits they might be Roche volume filling or even Roche volume overfilling
- Assumptions:
  - GCs have approx. constant angular momentum in an purely isothermal halo
  - Disk/Bulge contribution can be neglected



Thank you for your attention!