

MOCCA - code for simulation of large star cluster evolution

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Outline

- 1 Introduction to the Monte Carlo Method
- 2 The MOCCA Code
- 3 Dense Star Clusters
- 4 Conclusions



Basic Assumptions

- The Monte Carlo code is based on Hénon's (1971) implementation of the Monte Carlo method, which was substantially further developed by Stodółiewicz (1982).
- A star cluster is treated as a set of spherical shells which represent **an individual** object **star, binary ...** or a group of the same objects. Each shell is characterized by: mass (m), energy (E) and angular momentum (L).
- The relaxation process of a given object with all other objects in the system is approximated by only **ONE** interaction of two neighbouring shells - **Henon's trick**.

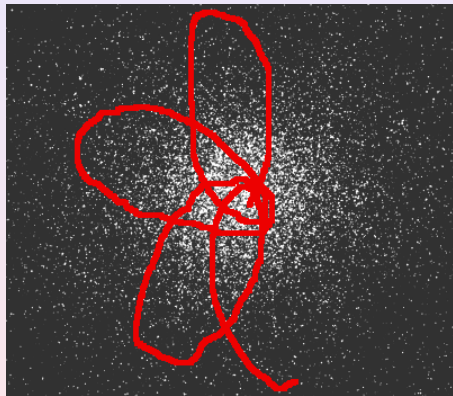
Actually, there are two independently developed Monte Carlo codes, both based on Hénon's method.

- Fred Rasio's Group + Mark Freitag
- Mirek Giersz's Group



Basic Assumptions

- Spherical symmetry - quick and easy potential computation in any place in the system.
- Dynamical equilibrium - star cluster is old, its age is much greater than the evolution time scale - no violent relaxation.
- In a specified potential, every star is characterized by its mass M , energy E and angular momentum L and moves on a rosette orbit - easy way of picking up the new object position.
- Evolution is driven by the relaxation process. The evolution time scale at each place in the system is proportional to the local relaxation time.



Basic Tricks

Statistical way of solving the Fokker-Planck equation.

- Instead of integrating a sequence of uncorrelated small-angle perturbations along the orbit, a single perturbation is computed at a randomly selected point on the orbit.
- Instead of considering the effect of all stars in the system, the perturbation is computed locally from a randomly chosen star, **in practise the nearest neighbour**.
- The computed single perturbation is multiplied by an appropriate factor in order to account for the cumulative effect of all small individual encounters with the rest of the stars in the system and for other points on the orbit.

If the procedure is correctly set up, the evolution of an artificial system is statistically the same as the evolution of a real one.

$$\sin^2\left(\frac{\beta}{2}\right) = 2\pi G^2 \frac{(m_1+m_2)^2}{w^3} n \Delta t \ln(\gamma N)$$



Free Parameters

Basic free parameters - only relaxation

- deflection angle, β .
- time step, τ - fraction of the local relaxation time.
- coefficient in the Coulomb logarithm - $\ln(\gamma N)$.

Free Parameters connected with physical processes implemented into the code

- Evolution of single and binary stars - Hurley 2000, 2002
- star escape criterion:
 - isolated system: all stars with positive energy escape **immediately**.
 - tidally limited: all stars with energy greater than critical energy escape **immediately** (Baumgardt 2001), or escape is **delayed** (Fokushige & Heggie 2000).
- cross sections for: binary formations, binary interactions or FewBody code for small N direct integration (Fregeau 2004).



FewBody Code Implementation - Advantages

All work connected with writing the interface between MOCCA and FewBody was done by Arkadiusz Hypki (PhD student).

ADVANTAGES

- Practically all channels of interactions for binaries are taken into account similarly as for N-body codes - **hierarchical triple, quadruples and higher hierarchies are exception - work in progress.**
- Exploration of formation channels for exotica in star clusters - BH, BS, CV, X-Ray binaries
- Realistic energy generation in binary interactions.
- Realistic description of the velocity changes of interacting objects allowing better tracking of binary spatial distributions.



Fokushige & Heggie Model - Advantages

- The escape process of a star from a tidally limited cluster is very complicated. To escape, it is not enough to have appropriate energy. A star has to find its way through the Lagrangian point L_1 . This needs time - the closer the energy of a star to the critical energy, the longer the time of escape - sometimes even several Gyrs.
- Some stars which have energy greater than critical energy (potential escapers), $E_c = -1.5GM/R_t$, can be backscattered to an energy smaller than the critical energy - they again become bound to the system. **The escape time scale is not proportional anymore to the half mass relaxation time.**
- Free parameter γ_{tid} , which adjusts the escape time scale.

ADVANTAGES

- **Escape process is not immediate anymore.**
- **There is no explicit N dependence - better scalability.**
- **Better description of a star cluster in the energy space.**



Interaction Probability

- The interaction probability depends on the interaction cross section σ

$$\sigma = \pi p^2 = \pi r_{pmax}^2 \left(1 + \frac{2Gm_{123}}{r_{pmax}V^2} \right), \quad (1)$$

- The number of interactions depend on r_{pmax} .
- Too large r_{pmax} means too distant interactions - practically no binary binding energy changes - very distant flybys. **Danger of repeating the relaxation step.**
- Too small r_{pmax} means too small energy generation in interactions.
- Theoretical value for $r_{pmax} \approx 2a$.



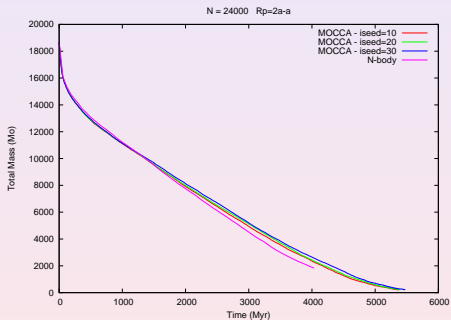
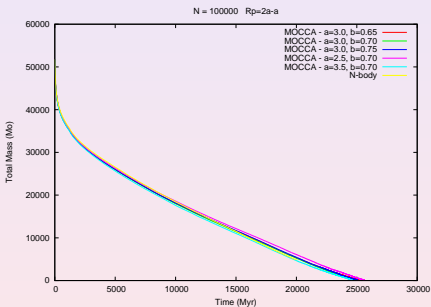
System Properties to Compare

Calibration of the MOCCA free parameters (y_{tid} , r_{pmax}) is done by comparison with N-body simulations for large systems (up to $N = 200000$ - [Jarrod Hurley's models](#)), for the following system properties.

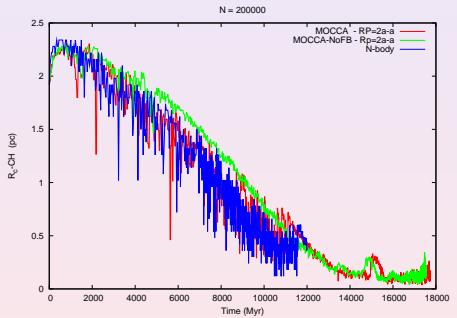
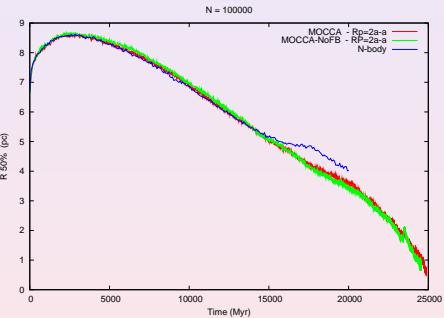
- Global parameters: cluster mass, half-mass radius, core radius, number of stars and binaries.
- Spatial distributions of binary binding energy, binary mass and number of binaries.
- Number of particular objects: BSS, BH



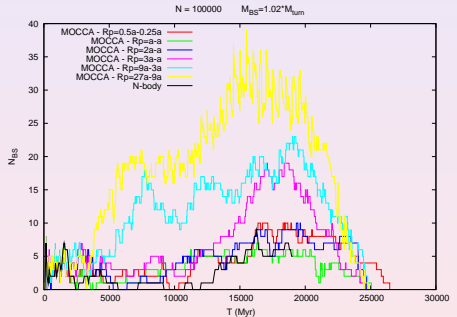
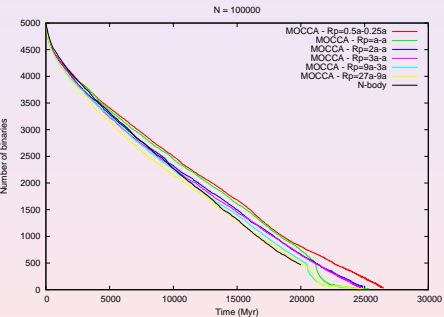
Star Cluster Mass



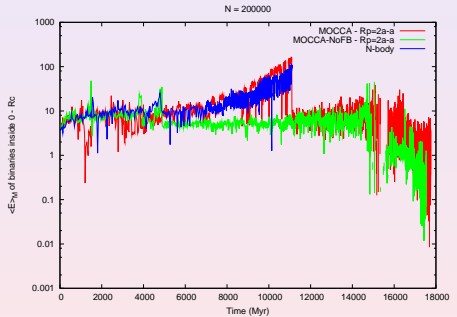
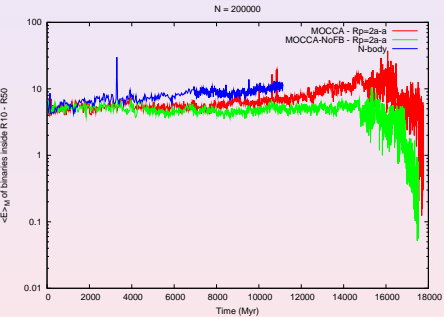
Core and Half-mass Radii



Number of Binaries and BSS

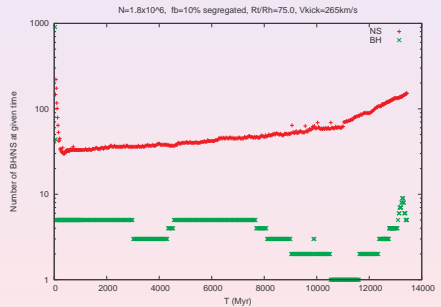
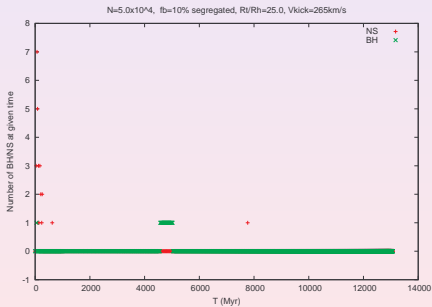


Binary Binding Energy Distribution

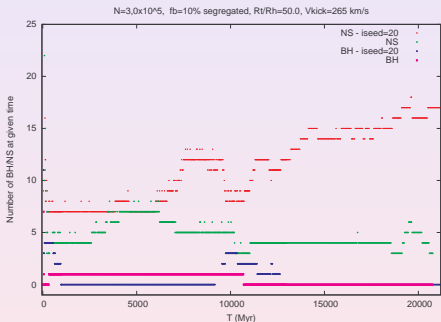


Formation of NSs and Bhs

Very **preliminary and byproduct** results of MOCCA simulations done for a project conducted with Nathan Leigh



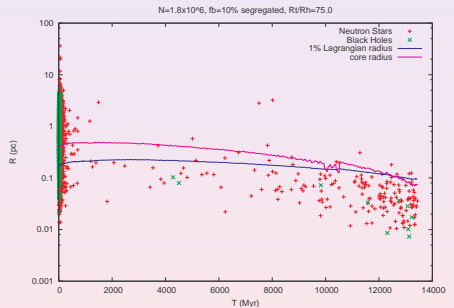
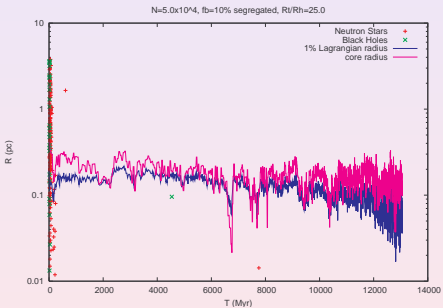
Formation of NSs and BHs



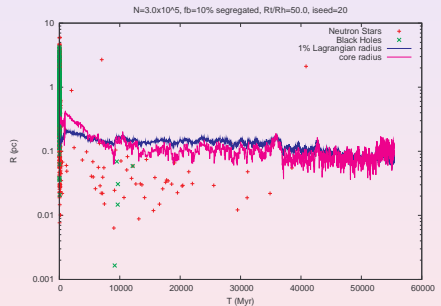
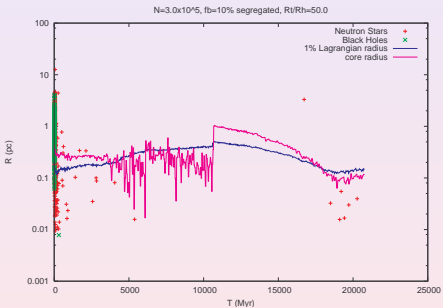
- BHs and NSs are formed from the most massive stars at the beginning of star cluster evolution. Well known and accepted processes of BHs and NSs formation.
- Substantial numbers of BHs and NSs are also formed during late stages of star cluster evolution! **Unexpected conclusion.**
- This number strongly depends on the cluster concentration and mass.
- The process of formation of BHs and NSs is strongly stochastic.



NS and BH Location in the Cluster



NS and BH Location in the Cluster



Processes Leading to BH and NS formation

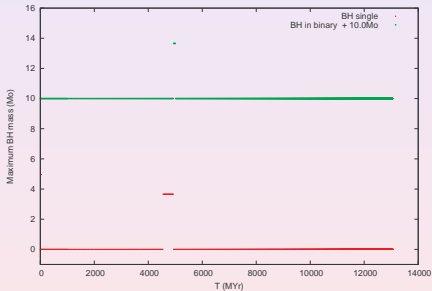
Cluster	object	evo-sing	evo-bin	coll	bin-sin	bin-bin	bin-form
$N = 5.0 \times 10^4$	NS	131	49	0	0	0	0
	BH	23	3	0	0	1	0
$N = 3.0 \times 10^5$	NS	799	198	1	2	2	0
	BH	127	25	0	0	1	93
$N = 3.0 \times 10^5$ <i>iseed</i> = 20	NS	781	236	4	27	6	13
	BH	159	34	0	6	0	5
$N = 1.8 \times 10^6$	NS	4818	1175	39	76	20	2
	BH	862	196	1	7	2	24

Table : Number of different types of interactions leading to BH or NS formation

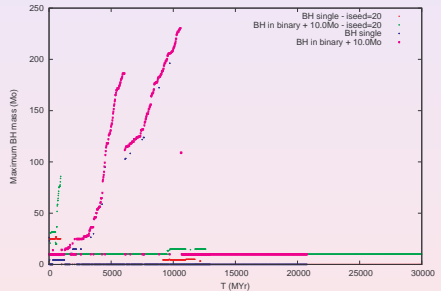


Mass of BH

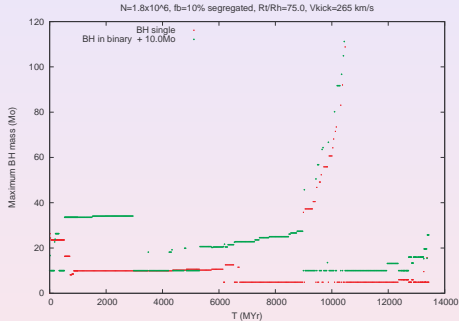
$N=5 \times 10^4$, $f_b=10\%$ segregated, $R_t/R_h=25.0$, $V_{kick}=265$ km/s



$N=3 \times 10^5$, $f_b=10\%$ segregated, $R_t/R_h=50.0$, $V_{kick}=265$ km/s



Mass of BH



The process of formation of very massive BH is very stochastic. Only 1/3 of all models (9 from 27) with the same initial conditions, but different statistical realisation show such grow of BH mass.

How to get very massive stellar BH?

- It seems, that to initiate the process of BH mass grow it is needed that **only one** BH is left in the system after the phase of the initial SN explosions (kicks or dynamical interactions).
- Formation of BH-any binary in 3-body interactions. This binary is the most massive object in the system. High formation probability, frequently BH-MS/RG.
- Interaction with other binaries and stars
 - orbit tightening leading to mass transfer from MS companion
 - exchanges and mergers, but binary is still there
 - "total" mergers - only BH is left, which very shortly will form a binary



Conclusions

- MOCCA code, similarly to N-body code, can directly follow few body interactions with the same level of detail and complexity.
- MOCCA code deals with escapers from tidally limited clusters in a similar way as N-body code does.
- MOCCA code is extremely fast - $N = 100000$ - about 2 hrs (N-body weeks). It is perfect for surveys of the initial condition parameter space.
- MOCCA code is able to provide information (as detailed as N-body code) about global cluster parameters and also about particular objects.
- MOCCA simulations of very concentrated clusters show the new possible way of formation of IMBH in globular clusters and show that BHs and NSs can form abundantly also during advanced phases of cluster evolution.
- MOCCA code is at present practically the only code which is able to quickly and reliably simulate the evolution of real, large and old star clusters and provide a detailed reconstruction of observational data.

