Symmetries and dynamics of star clusters

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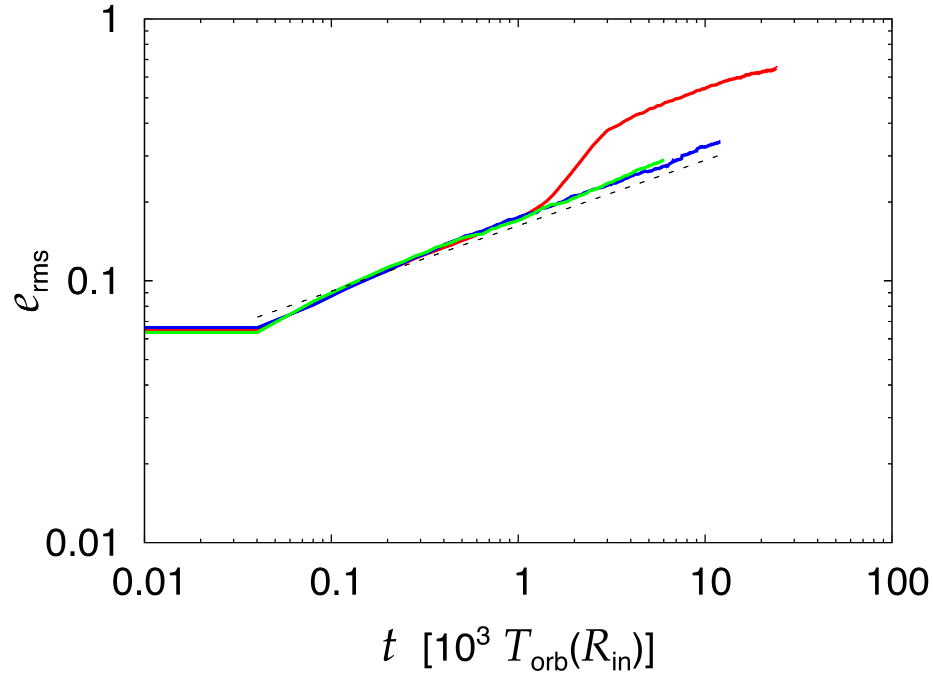
Motivation: galactic nuclei

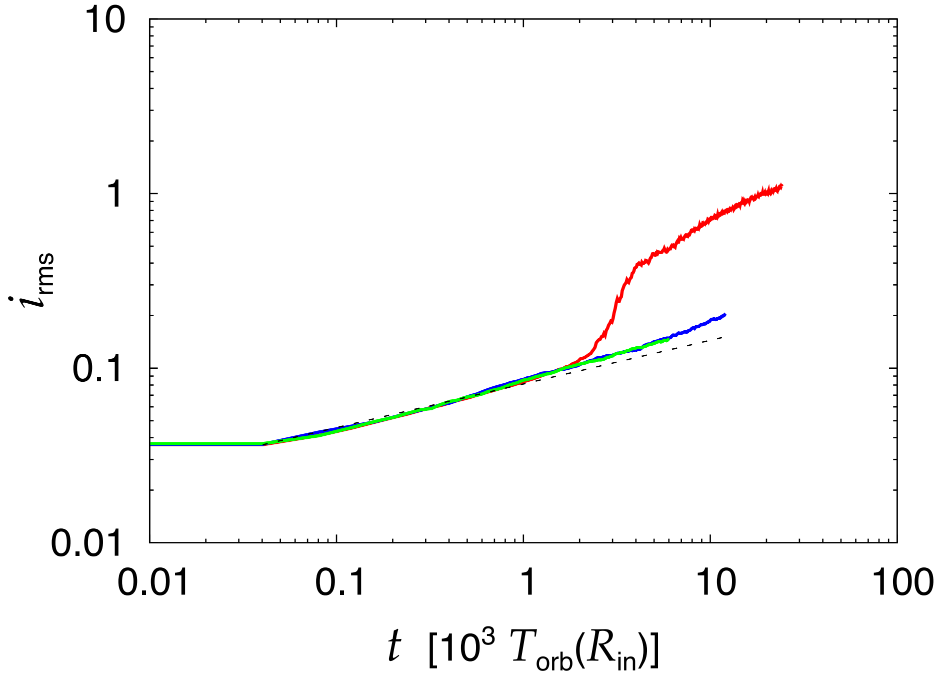
- Supermassive black hole (SMBH)
- Old spherically symmetric star cluster
  - dynamical evolution of such clusters: Peebles (1972a,b), Bahcall & Wolf (1976, 1977) . . .
- Massive axisymmetric gaseous torus
- Surprise: young stellar disc
  - Levin & Beloborodov (2003): Milky Way
    Bender et al. (2005): M31
  - dynamical evolution of such disc(s): Nayakshin et al. (2006), Cuadra et al. (2008), Löckmann et al. (2009), Šubr et al. (2009), Kocsis & Tremaine (2011) . . .
  - let's broaden these analyses

Part 1: Coupling of the disc and the cluster

- SMBH
  - predefined Keplerian potential (mass $M_\bullet$)
- Stellar disc
  - particles ($N \sim 10^3,\ m \sim 10^{-6} - 10^{-5}\, M_\bullet,\ R \in \langle R_{\rm in}, R_{\rm out} \rangle$)
  - initially thin and circular
- Spherical star cluster
  - predefined analytic potential ($M_{\rm c}(R_{\rm out}) \sim 0 - 1\, M_\bullet$)
  - particles ($N \sim 10^4 - 10^5,\ R \in \langle R_{\rm in}/2, 2R_{\rm out} \rangle$)
- Numerical N-body integrator NBODY6 (Aarseth 2003)

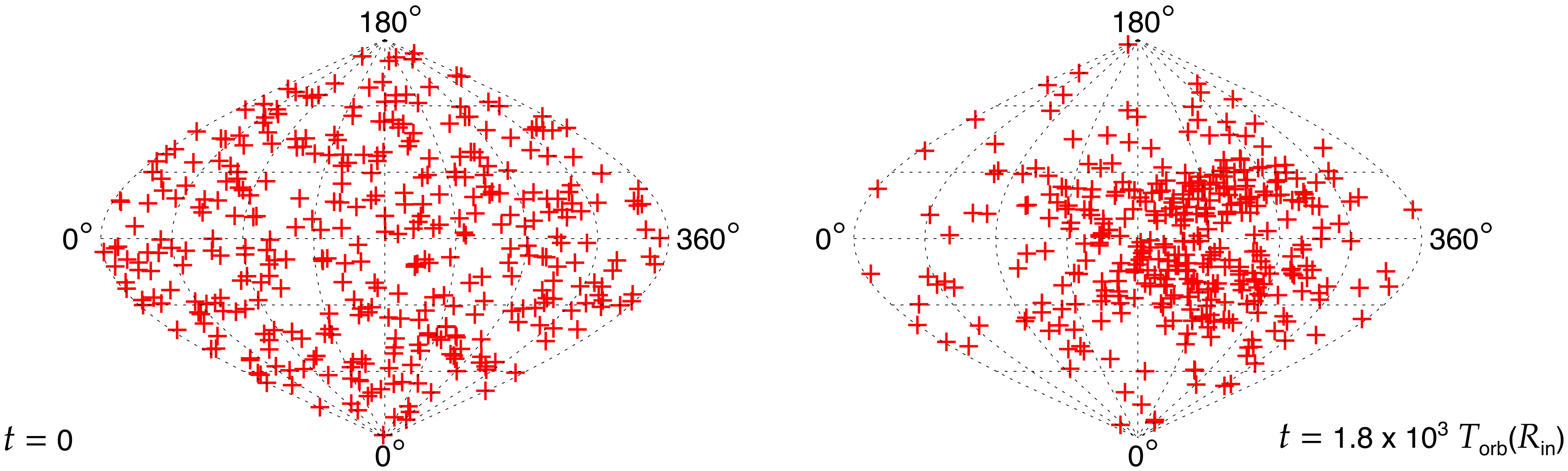
RMS eccentricity and inclination in the disc





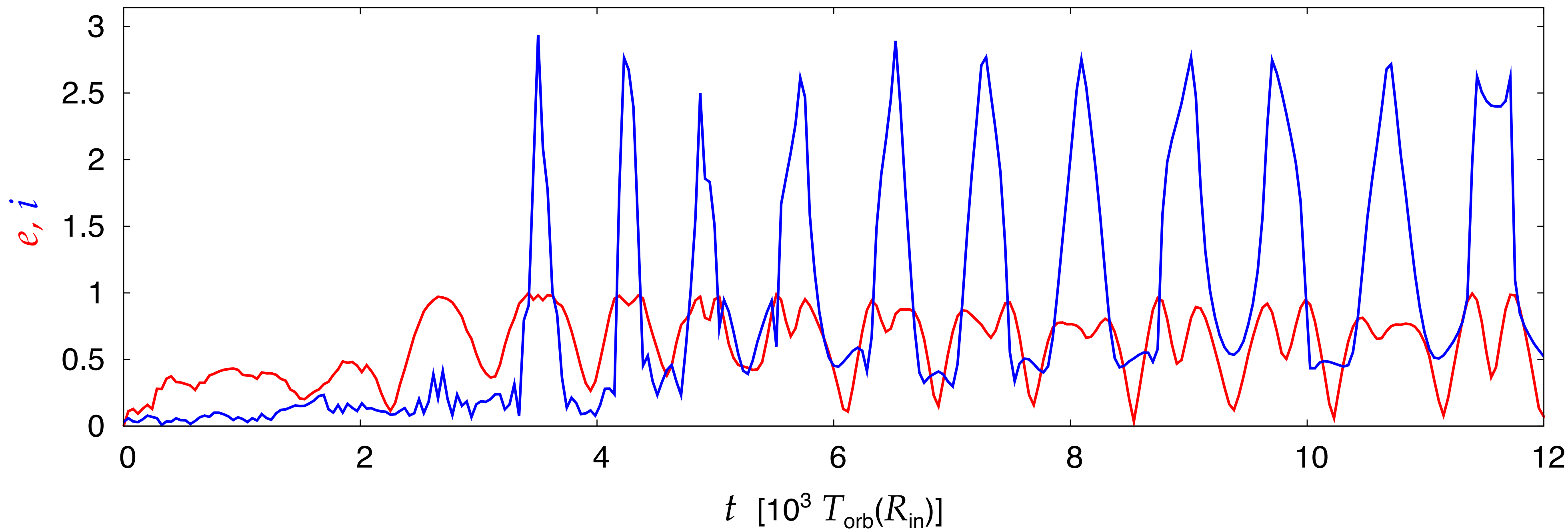
- Isolated disc: in accord with theoretically predicted $\propto t^{1/4}$ (dashed)
- Analytic cluster: no effect
- N-body cluster: accelerated evolution (Haas et al. 2012)

Eccentricity vector distribution in the cluster



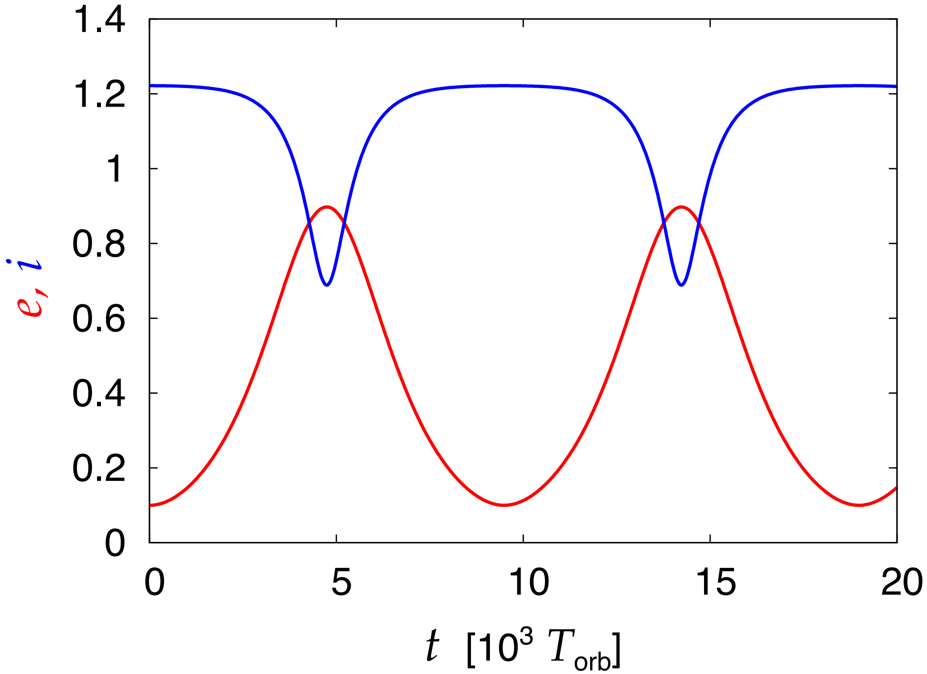
- Orbits from the innermost parts of the cluster displayed $(a < 1.5\, R_{\rm in})$
- Left: initially spherically symmetric
- Right: non-spherical structure in the evolved state

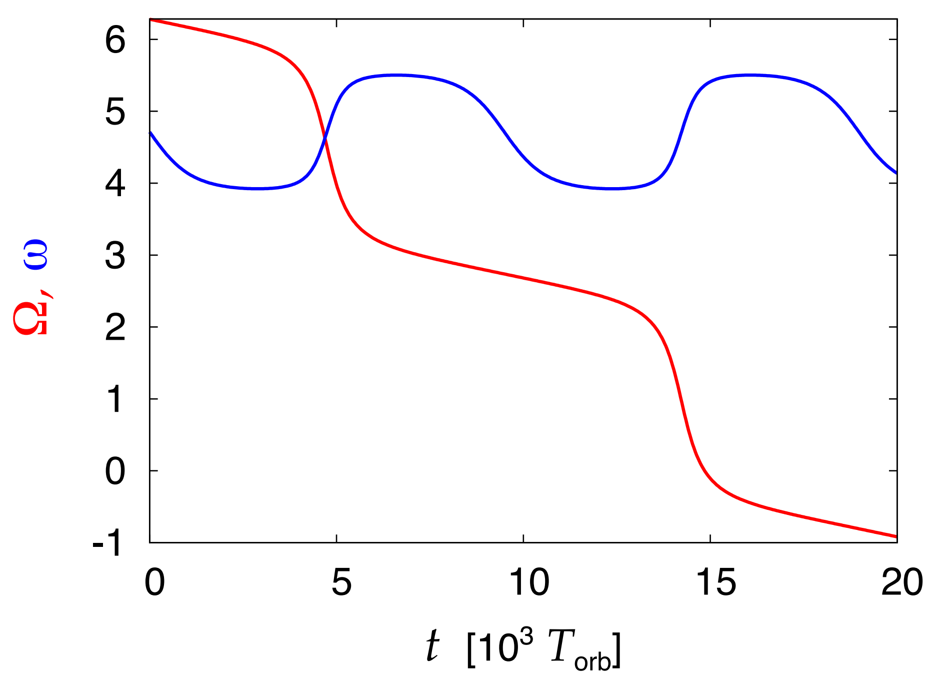
Extremely oscillating orbits in the disc



- Non-spherical structure: flattened overdensity perpendicular to the disc
- Kozai-Lidov mechanism (Kozai 1962, Lidov 1962)
- Transformation: $\Omega|_{\rm structure} \longrightarrow i|_{\rm disc}$
- Averaging of oscillating orbits $\Rightarrow$ accelerated growth of $e_{\rm rms}$ and $i_{\rm rms}$

Kozai-Lidov mechanism





- SMBH
  - Keplerian orbits: a, e, i, Ω, ω
- SMBH + axisymmetric perturbation
  - Kozai-Lidov oscillations of e, i and precession of Ω, ω
  - with respect to the plane of symmetry of the perturbing potential

Conclusions of part 1

- Orbital evolution of the stellar disc around the SMBH embedded in the predefined analytic spherical potential is similar to that of the isolated disc
- Initially spherically symmetric N-body star cluster develops a non-spherical structure due to the potential of the disc
- Subsequently, this structure causes extreme oscillations of e and i of the orbits in the disc
- High eccentricities are important for many astrophysical processes
  - production of hyper-velocity stars and S-stars (Löckmann et al. 2008, Bromley et al. 2012)
  - tidal disruption of stars and their feeding to the SMBH (Karas & Šubr 2007)
  - generation of gravitational waves (Berry & Gair 2012)

Part 2: Coupling of near-Keplerian orbits

- Two stars in the potential of SMBH, ring and spherical cluster
- Evolution of the orbits? (Haas et al. 2011b):

$$\frac{\mathrm{d}\cos i}{\mathrm{d}t} = -\frac{1}{mna^2}\frac{\partial\overline{\mathcal{R}}}{\partial\Omega}\,, \qquad \frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{1}{mna^2}\frac{\partial\overline{\mathcal{R}}}{\partial\cos i}$$

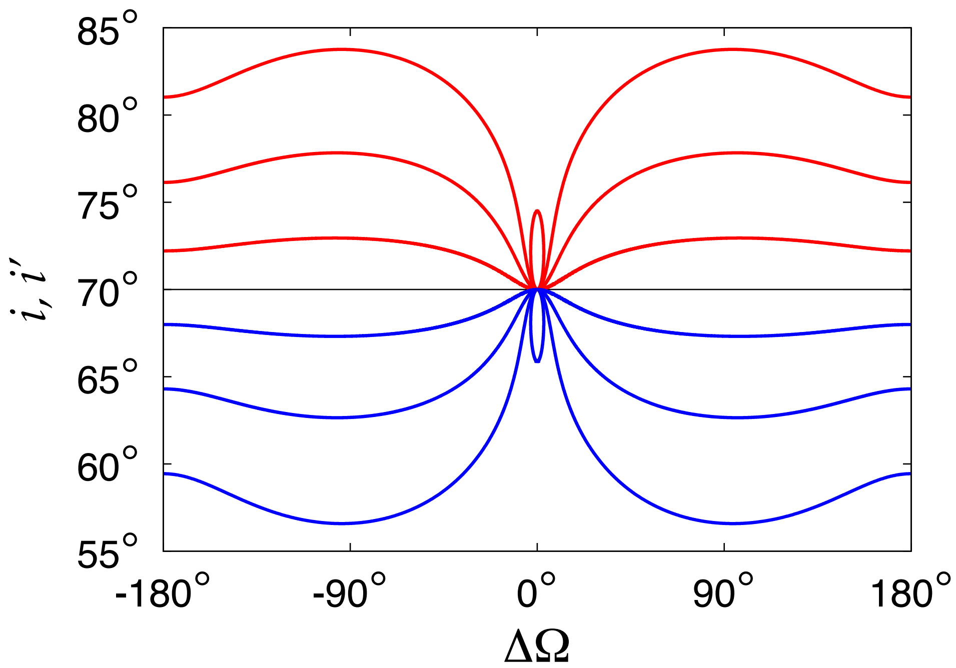
$$\overline{\mathcal{R}} = \overline{\mathcal{R}}_{\mathrm{r}} + \overline{\mathcal{R}}_{\mathrm{r}}' + \overline{\mathcal{R}}_{\mathrm{i}}$$

$$\overline{\mathcal{R}}_{\mathrm{r}} = -\frac{GmM_{\mathrm{r}}}{R_{\mathrm{r}}}\,\Psi\left(a/R_{\mathrm{r}}, \cos i\right)$$

$$\overline{\mathcal{R}}_{\mathrm{i}} = -\frac{Gmm'}{a}\,\Psi\left(\alpha, \boldsymbol{n}\cdot\boldsymbol{n}'\right)$$

- Analogic equations for the second star i', Ω'

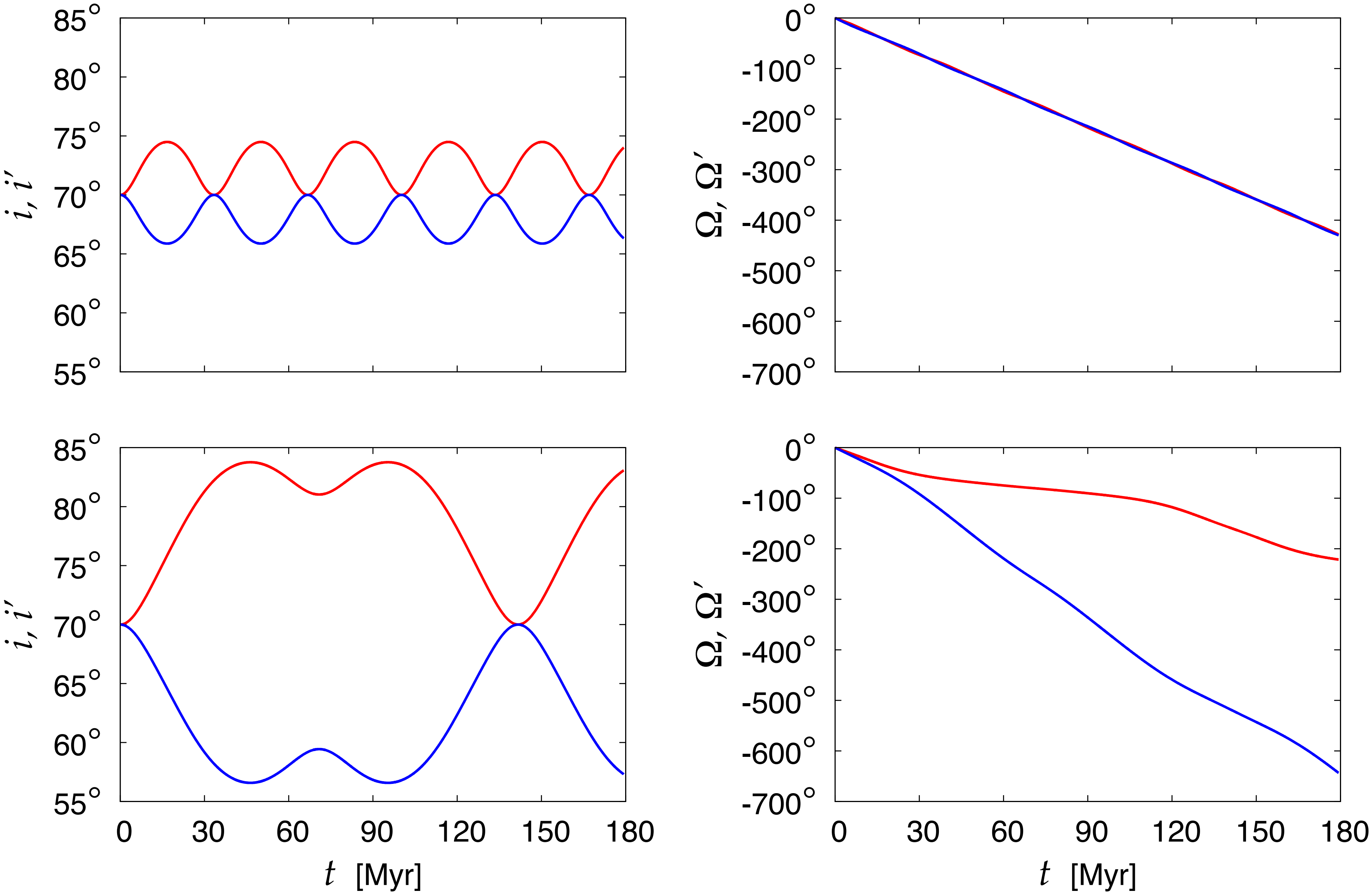
Integrals of motion $\Rightarrow$ 2 modes of evolution



- Total energy
- Z-component of the total angular momentum: $m \cos i + m' \alpha^{1/2} \cos i'$
- $\overline{\mathcal{R}}$

$\Rightarrow$ isolines of $\overline{\mathcal{R}}$ for different masses m, m'

Numerical solutions



Generalisation for multiple stars

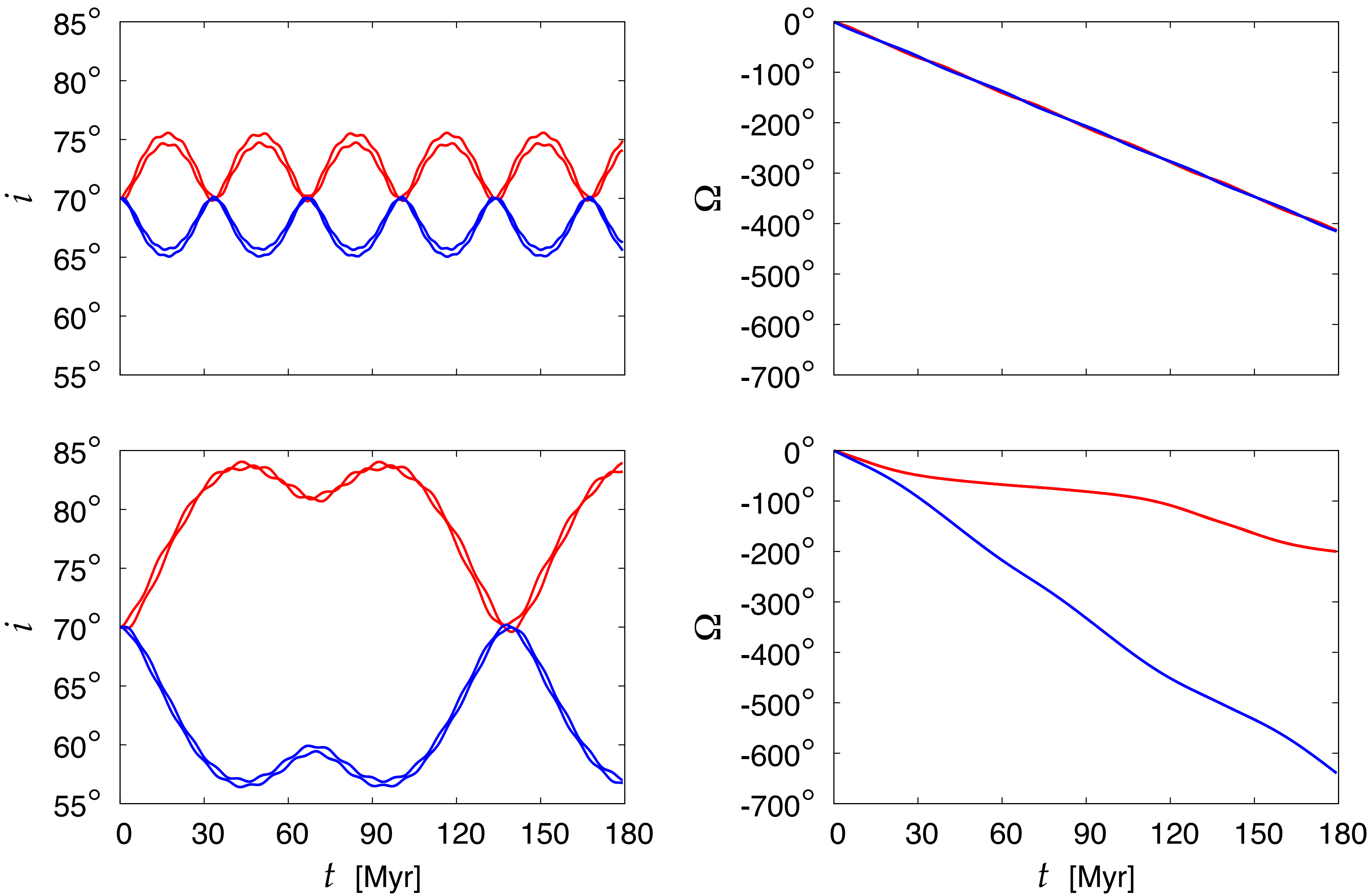
- Sumation over the individual stars

$$\overline{\mathcal{R}}_{\mathrm{r}} = -\sum_{k} \frac{G m_k M_{\mathrm{r}}}{a_k} \, \Psi \left(a_k / R_{\mathrm{r}}, \boldsymbol{n}_k \cdot \boldsymbol{e}_z \right)$$

$$\overline{\mathcal{R}}_{\mathrm{i}} = -\frac{1}{2} \sum_{k \neq l} \frac{G m_k m_l}{a_{kl}} \, \Psi \left(\alpha_{kl}, \boldsymbol{n}_k \cdot \boldsymbol{n}_l \right)$$

- Complex evolution in a general case
- The impact of a group of strongly interacting stars upon the rest of the system similar to that of one suitable star

Multiple stars: simple solutions



Conclusions of part 2

- We have developed a semi-analytic model for a system of N gravitationaly interacting stars in the dominating potential of the SMBH which is perturbed by an extended spherically symmetric star cluster and a distant axisymmetric source (Haas et al. 2011b)
- Mutual interaction of the stars may lead to dynamical coupling of their orbits
- In the interaction with the rest of the system, the coupled orbits effectively act as a single orbit of suitable parameters

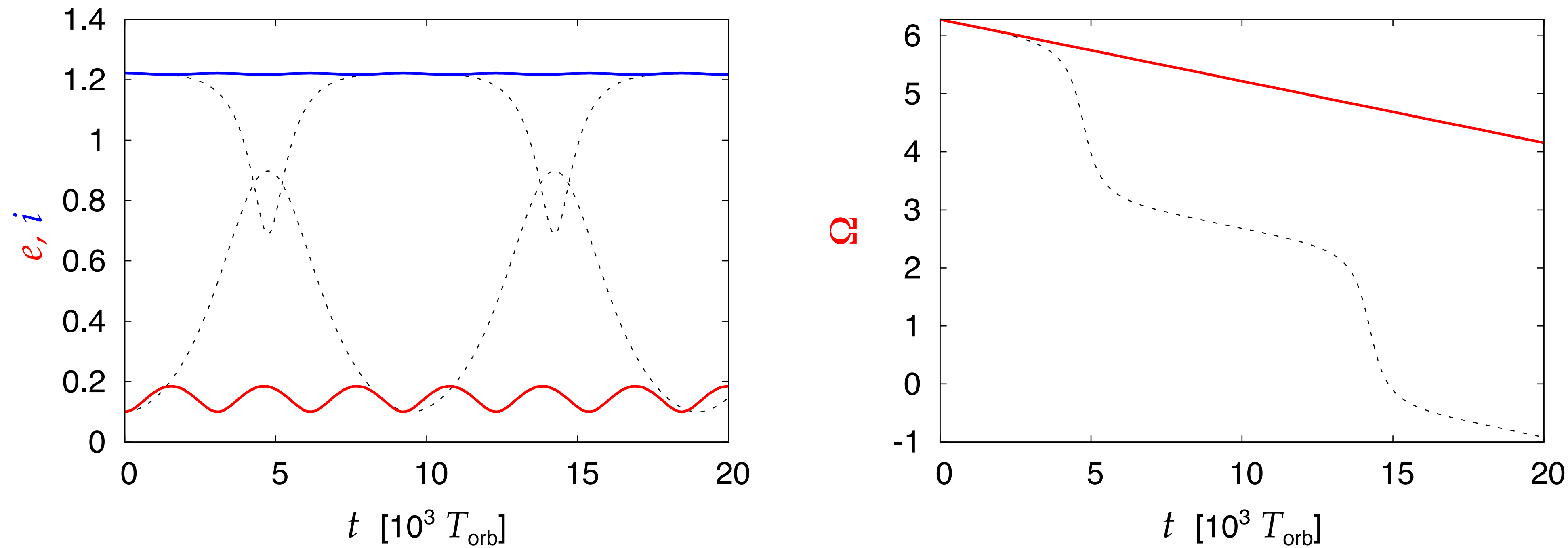
Part 3: Innermost two parsecs of our Galaxy

- SMBH: compact radio source Sgr A*
  - mass $\approx 4\times10^6\,M_\odot$; distance from the Sun $\approx$ 8 kpc (Eisenhauer et al. 2005, Gillessen et al. 2009)
- 177 young stars (Paumard et al. 2006, Bartko et al. 2009, 2010)
  - most of them $\approx$ 0.03 – 0.5 pc from the SMBH; age $\approx$ 6 Myr
  - rougly one half of these form a disc — clockwise system (CWS)
  - the remaining scattered around the CWS
- Roughly spherical cluster of old stars (Schödel et al. 2007)
- Massive gaseous torus — circumnuclear disc (CND)
  - radius $\approx$ 1.8 pc; upper estimate of the mass $\approx 10^6\,M_\odot$ (Christopher et al. 2005)
- CND roughly perpendicular to the CWS (Paumard et al. 2006)

Origin of the young stars?

- Tidal forces of the SMBH $\Rightarrow$ standard star formation impossible
- Levin & Beloborodov (2003)
  - fragmentation of a self-gravitating gaseous disc $\Rightarrow$ CWS
  - stars observed outside the CWS?
- Hobbs & Nayakshin (2009), Löckmann & Baumgardt (2009)
  - interaction of two discs (CWS, CCWS) $\Rightarrow$ ok
  - special initial conditions needed (t_0, i_0^{CWS}, i_0^{CCWS})
- Šubr et al. (2009)
  - spherical cluster and CND included $\Rightarrow$ deformation of the CWS, ok
  - mutual interaction of the stars neglected
  - special initial conditions needed ($i_0^{\mathrm{CWS}} \approx 90^\circ$)
- Self-gravity of the stars? (Šubr 2011, Haas et al. 2011a)

Damped Kozai-Lidov mechanism



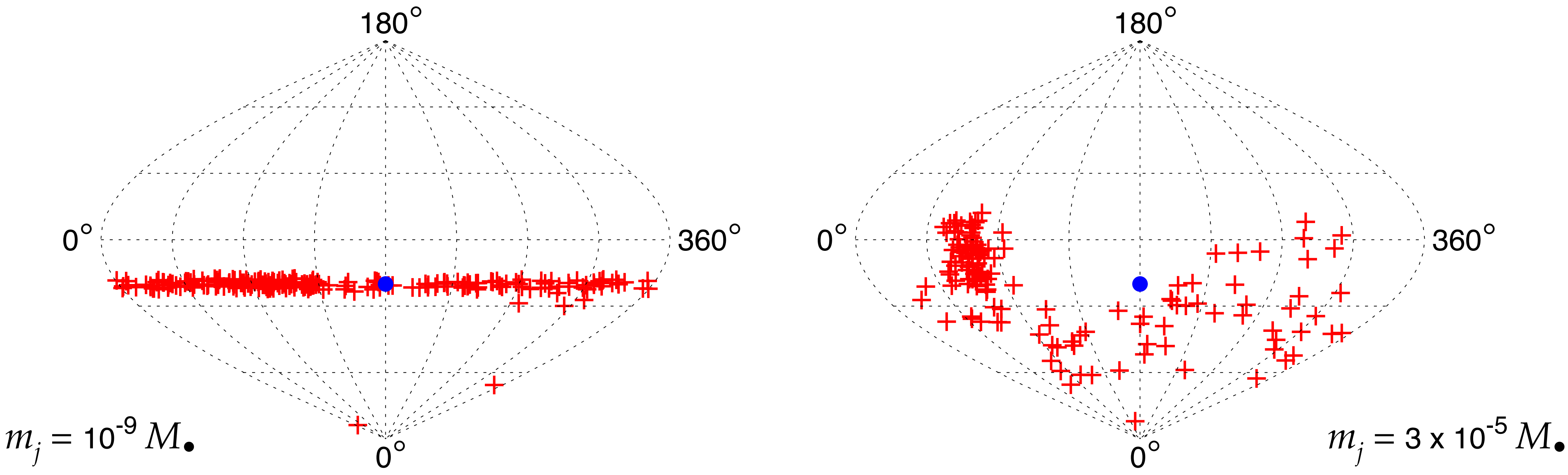
- SMBH + CND + spherical cluster
  - fast rotation of $\omega \Rightarrow$ oscillations of e and i damped
  - differential precession of Ω

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} \propto -\frac{1+\frac{3}{2}e^2}{\sqrt{1-e^2}}a^{3/2}\cos i$$

Numerical model

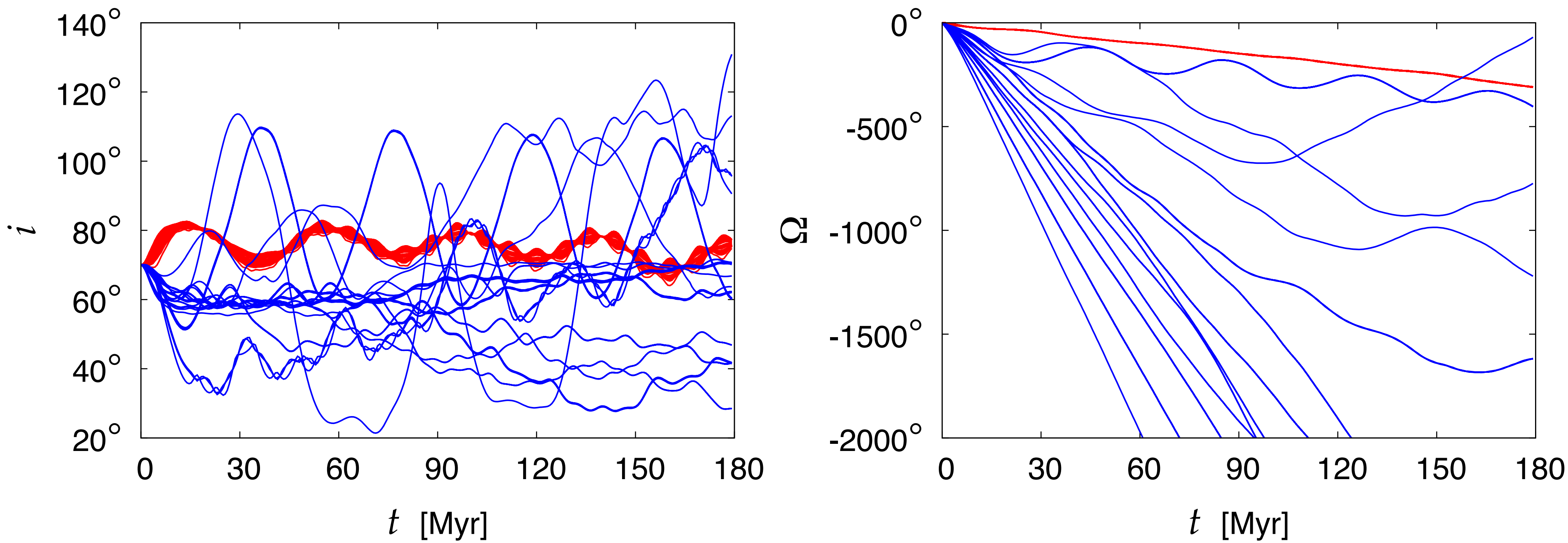
- SMBH
  - predefined Keplerian potential (mass $M_\bullet$)
- CND
  - massive particle ($M_{\rm CND} \sim 0.1 - 1\,M_\bullet$)
- CWS
  - particles ($N \approx 200,\ m \sim 10^{-7} - 10^{-5}\,M_\bullet$)
- Spherical star cluster
  - predefined analytic potential ($M_{\rm c}(R_{\rm CND}) \sim 0 - 1\,M_\bullet$)
- NBODY6 (Aarseth 2003)

Angular momentum distribution in the disc



- Blue circle: initial state ($i \approx 70^\circ$)
- Red crosses: $t = 6$ Myr
- Compact group at $i \approx 90^\circ$ in the right panel: CWS

Application of the semi-analytic model



- Coherently evolving subset of orbits whose inclination initially increases: CWS

Conclusions of part 3

- All of the young stars in the Galactic Centre observed at distances $\approx$ $0.03 - 0.5$ pc from the central SMBH may have been born within a single gaseous disc (Šubr 2011, Haas et al. 2011a)
- Subsequently, these stars may have been brought to their present locations by the effects of dynamics in the combined potential of the SMBH and the CND
- Scanning of the parameter space of our numerical model reveals that the observed configuration is reproduced for parameters which are in accord with the observational constraints
- The suggested scenario is described by a simple semi-analytic model (Haas et al. 2011b)