Re-Winding the Dynamical Clocks of Star Clusters

Nathan Leigh (ESA)

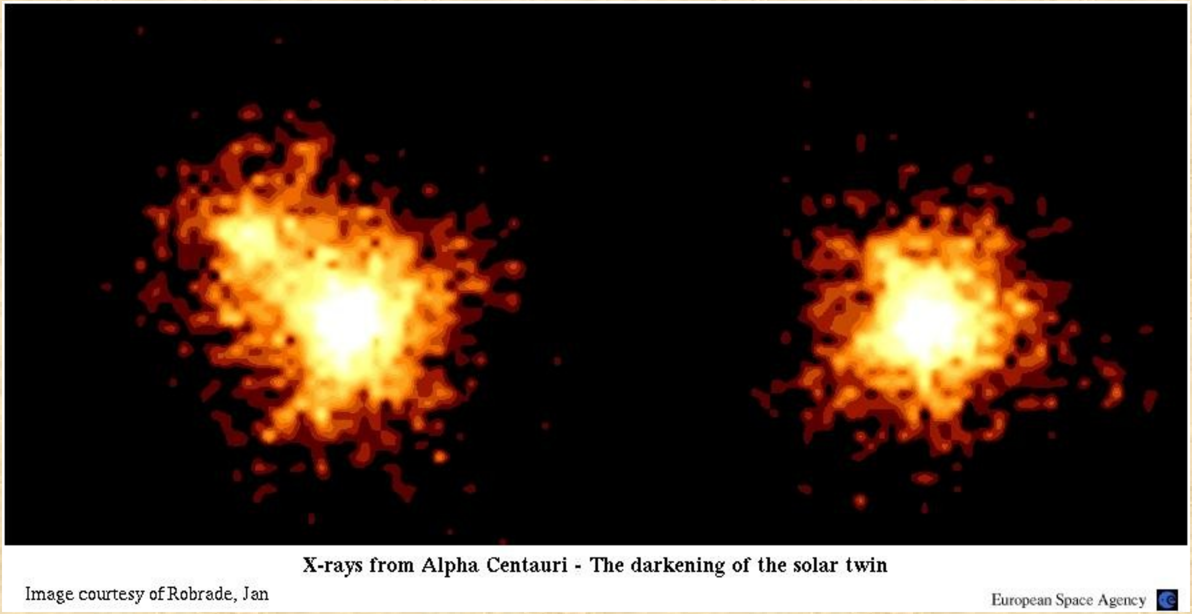
December 3, 2012

Gluehwine Aarseth N-body Meeting

High-Order Multiplicity in Stellar Dynamics

Multiplicity in the Milky Way

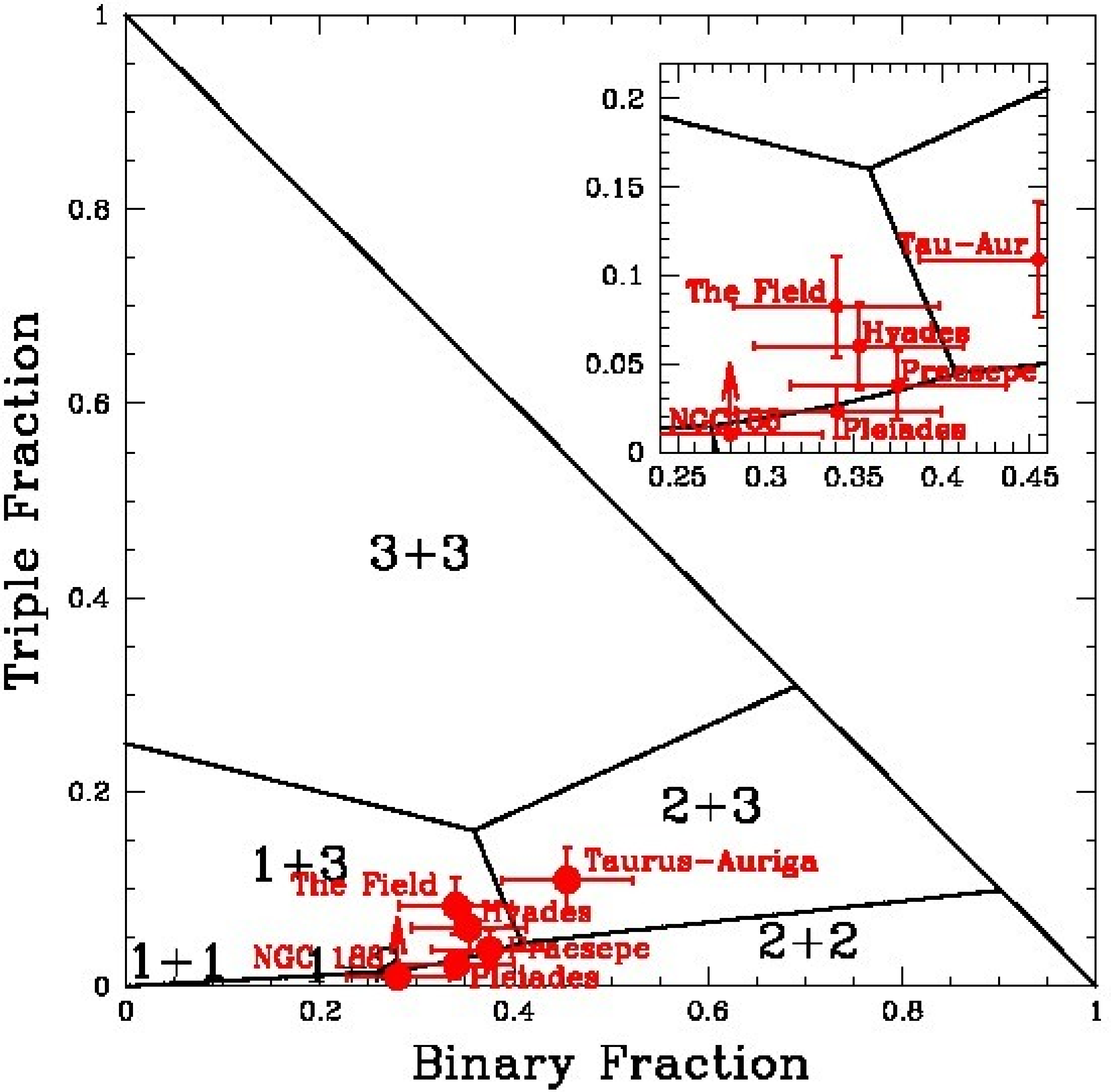
- Multiple star systems, including high-order multiples, are common in our Galaxy
- This is now known to be the case for:
  - The Field
  - Young star-forming regions
  - Open clusters
  - Dense globular clusters



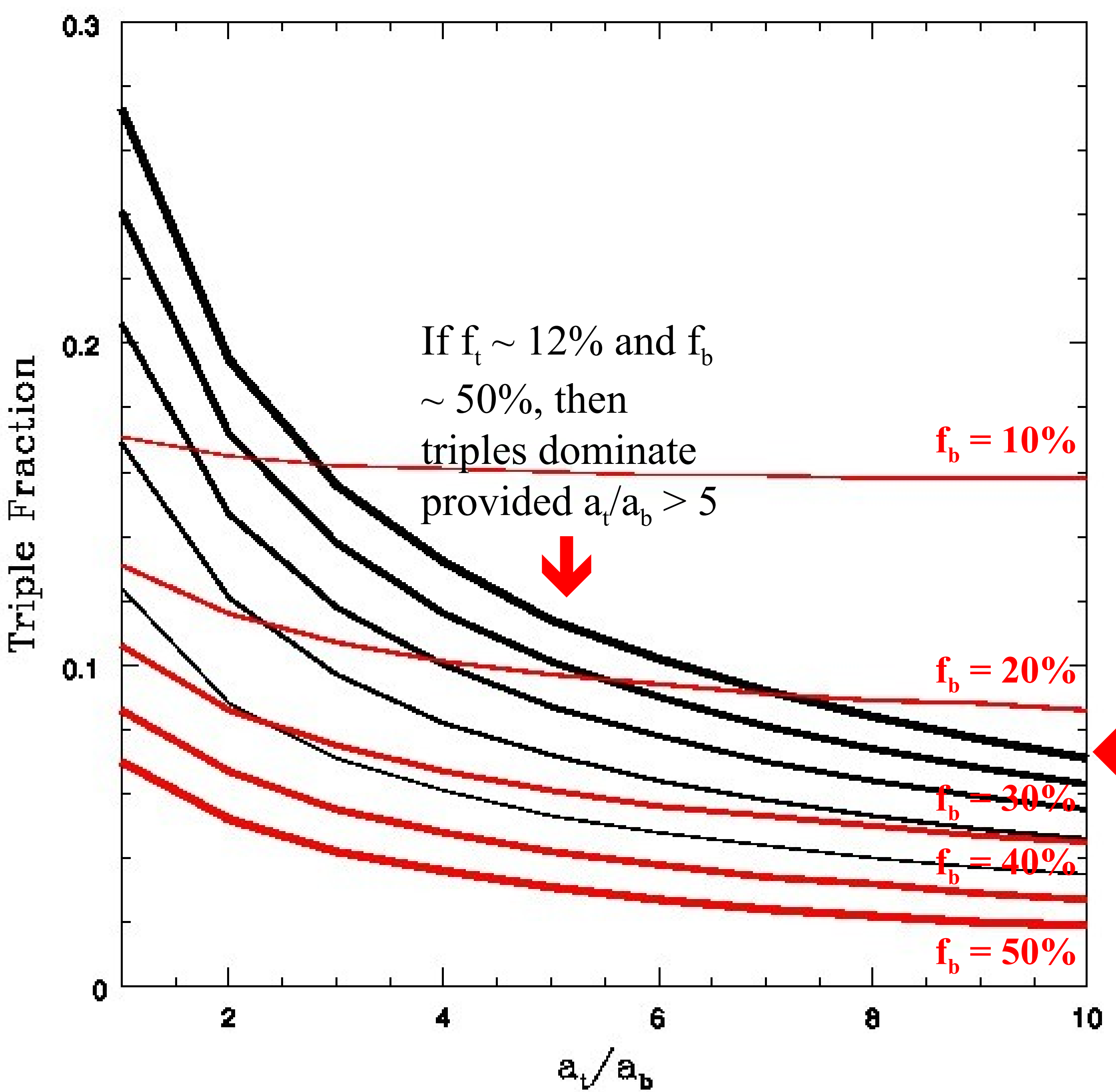
X-rays from the triple star system Alpha Centauri. Image courtesy of Jan Robrade (ESA).

The Dynamical Implications of Multiplicity?

Questions: What is the dynamical significance of the presence of high-order multiples in dense stellar environments? How can they be used as "dynamical clocks"?



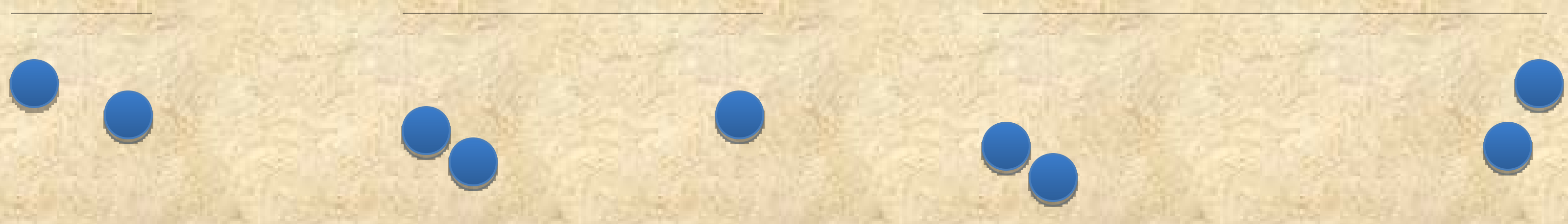
For surprisingly small numbers of triples, they can be undergoing dynamical encounters as frequently as either single or binary stars.



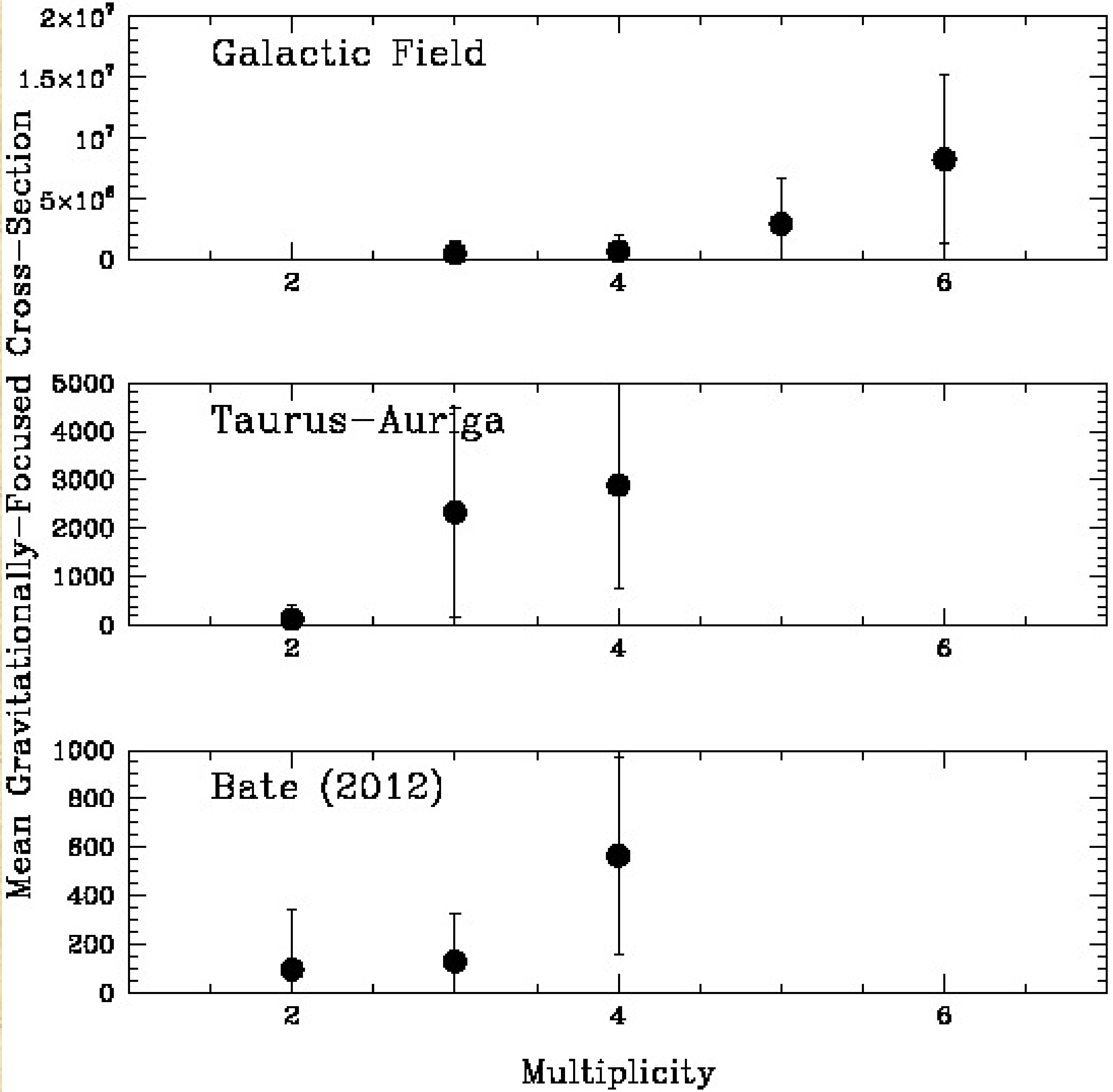
If $f_t \sim 7\%$ and $f_b \sim 50\%$, then triples dominate provided $a_t/a_b > 10$

Key Point

The average cross-section for collision increases with increasing multiplicity.



•In order to satisfy the requirements for long-term dynamical stability, the maximum orbital separation tends to increase with increasing multiplicity



Increasing the gravitationally-focused cross-section *reduces* the time between encounters. In turn, this increases the dynamical significance of high-order multiples.

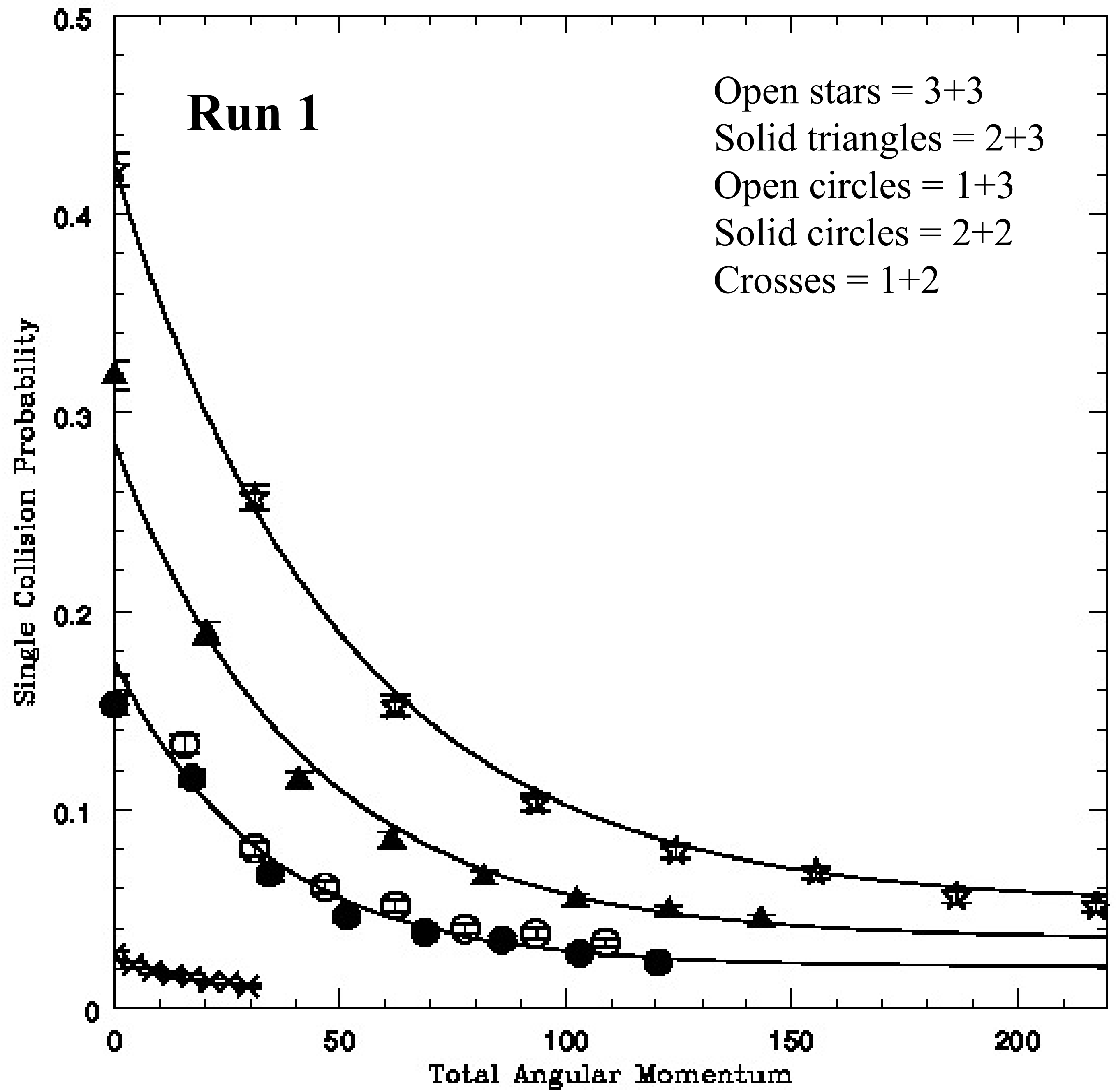
Caveat

The probability that a direct physical collision will occur between any two stars during a dynamical encounter increases with increasing multiplicity.

- Multiplicity could be important for:
  - The *destruction* of compact binaries
  - The formation of stellar exotica (e.g. blue stragglers)

N-Dependence for the Collision Probability

- Choose a general functional form for the collision probability
- This function consists of:
  - A power-law dependence on N
  - An exponential term that drops off with increasing:
    - » Angular momentum L
    - » Number of objects N
  - An energy term in the form of the semi-major axis of the shortest-period orbit
- Fit this function to the results of our scattering experiments, and obtain the best-fit parameters



Run 1
Open stars = 3+3
Solid triangles = 2+3
Open circles = 1+3
Solid circles = 2+2
Crosses = 1+2
Single Collision Probability
Total Angular Momentum
0
0.1
0.2
0.3
0.4
0.5
0
50
100
150
200

Results

Parameter	Run 1	Run 2	Run 3
α	0.16 ± 0.02	1.56 ± 0.24	2.15 ± 0.52
β	2.19 ± 0.09	1.93 ± 0.09	2.33 ± 0.15
δ	-0.12 ± 0.01	-0.09 ± 0.01	-0.17 ± 0.02
γ	0.14 ± 0.01	0.14 ± 0.01	0.23 ± 0.03

The probability of a direct collision occurring between any two stars during a dynamical interaction scales with the number of objects involved as N^2.[1]

[1]Probably should be N(N-1), or "N choose 2".

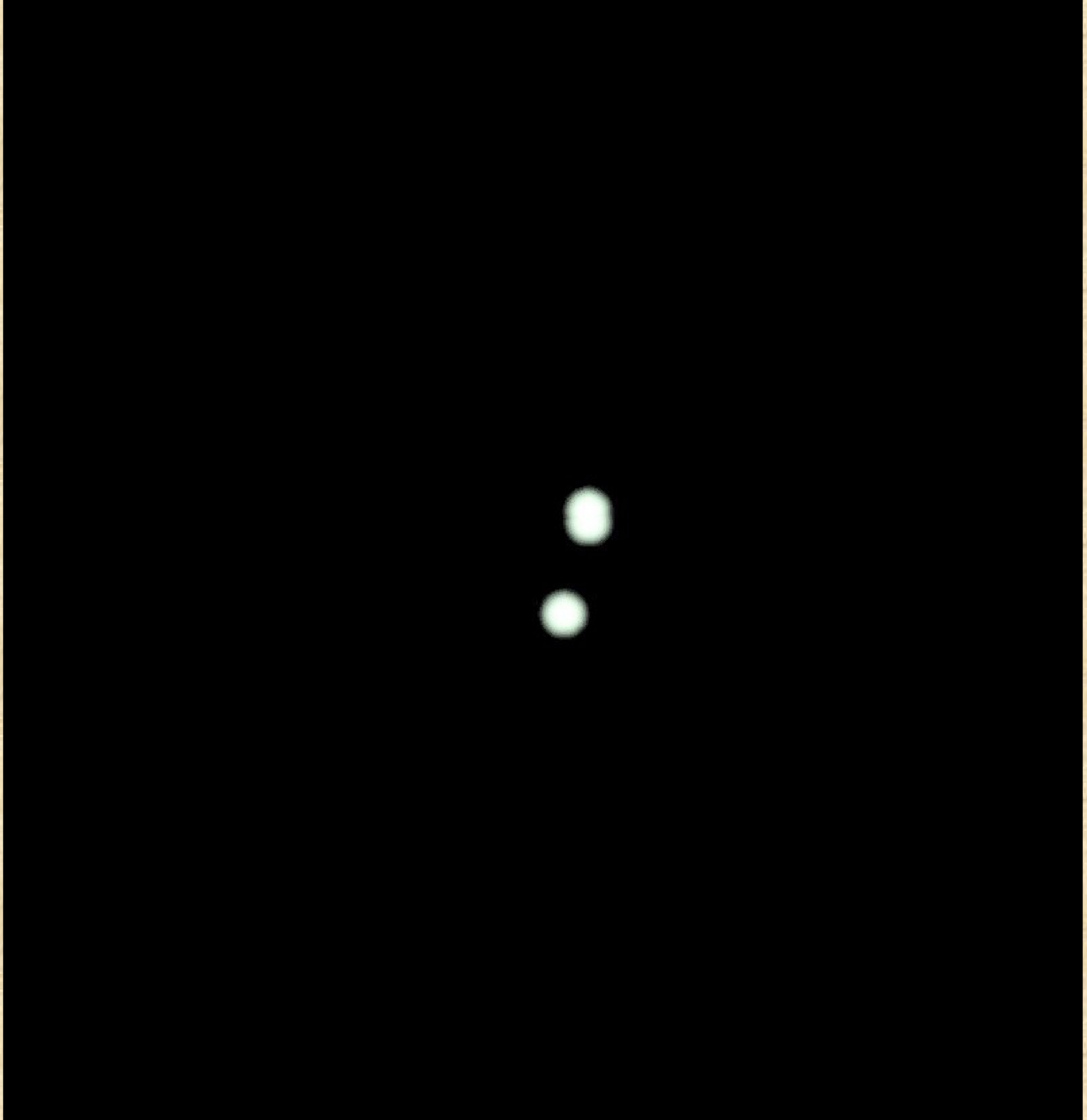
Interpretation?

- Consider two possibilities:

  1. $N_{cc} \sim N^2$ with N_{cross} = constant
  2. $N_{cross} \sim N^2$ with N_{cc} = constant

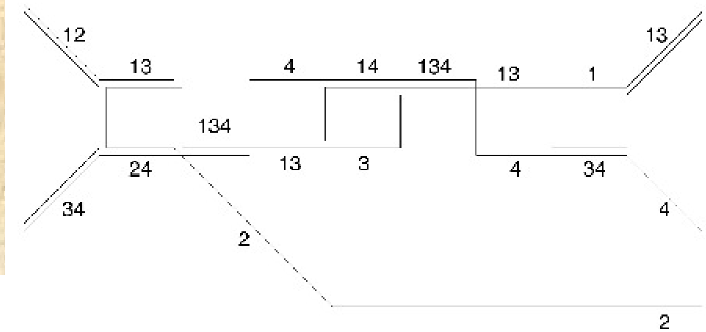
- In the limit of very large N, we have $\mathbf{\Gamma_{coll} \sim N^2}$, where Γ_{coll} is the collision rate derived from the **mean-free path approximation**

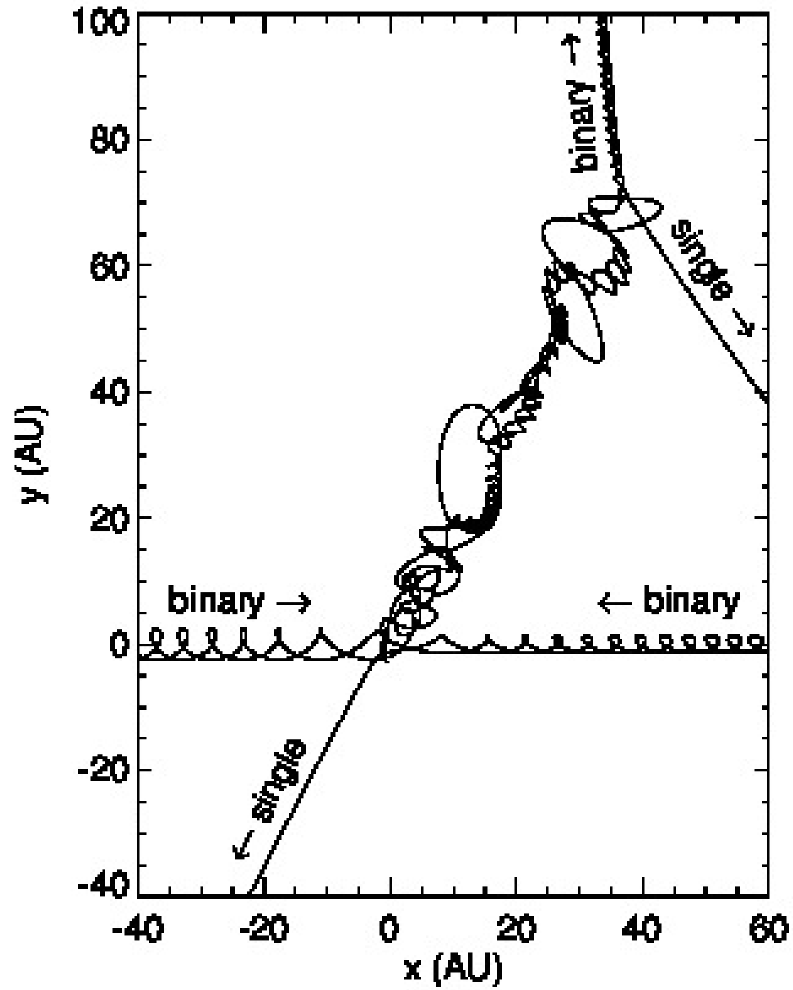
  ➔ Coincidence?

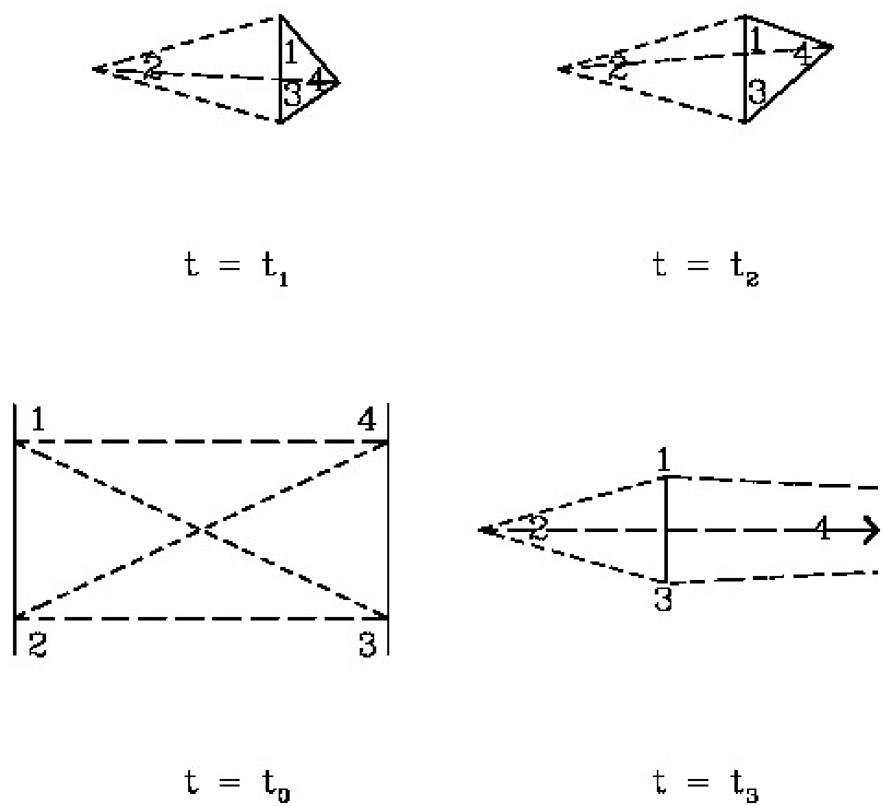


Movie credit:
Aaron Geller
(Northwestern
University)

Future Work
(see Appendix in Leigh & Geller 2012)



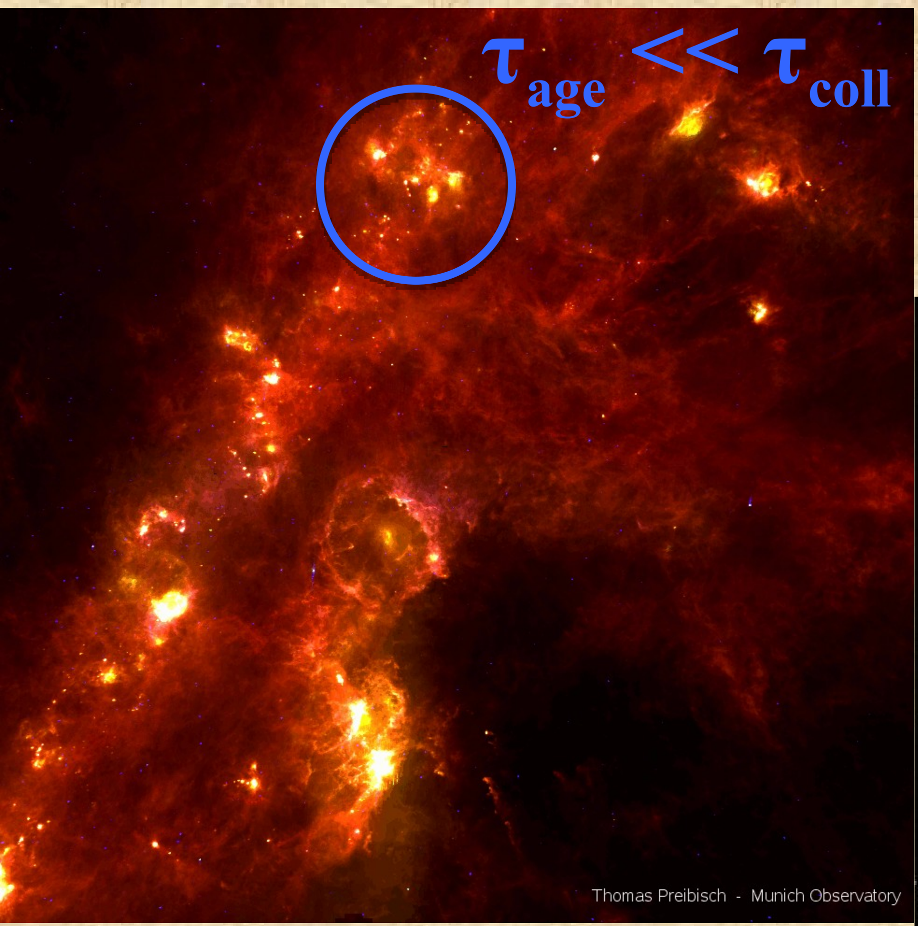




Dynamical Age-Dating

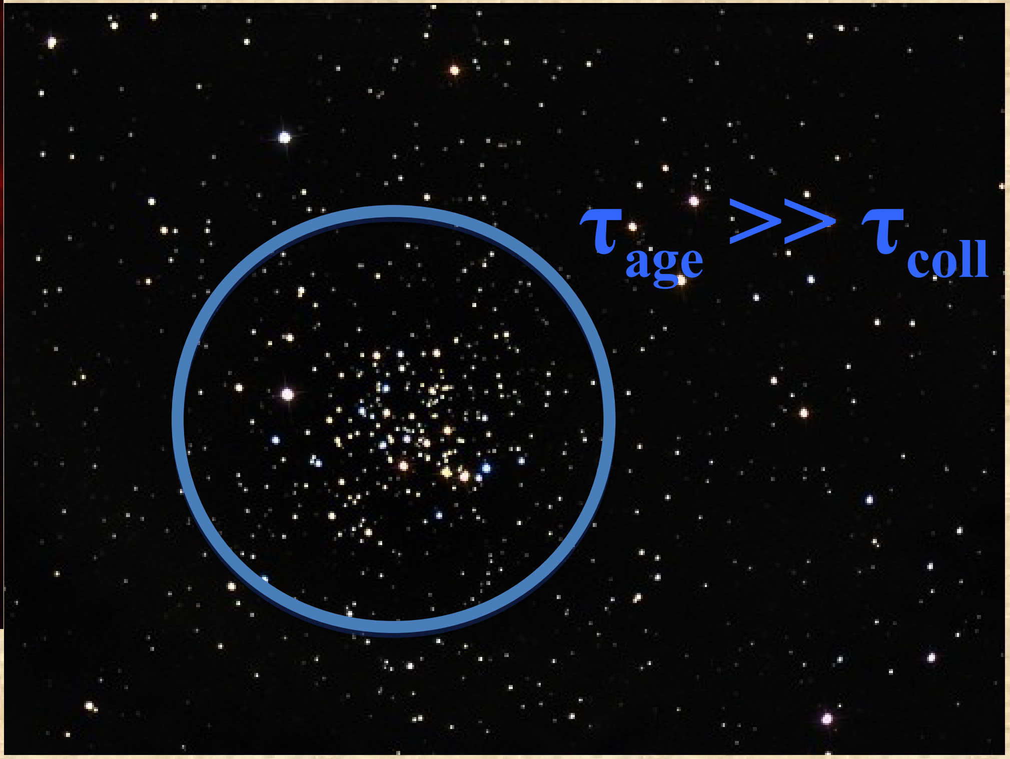
Question: How can you distinguish between ***primordial*** versus ***dynamically-processed*** populations of binary and triple star systems in clusters?

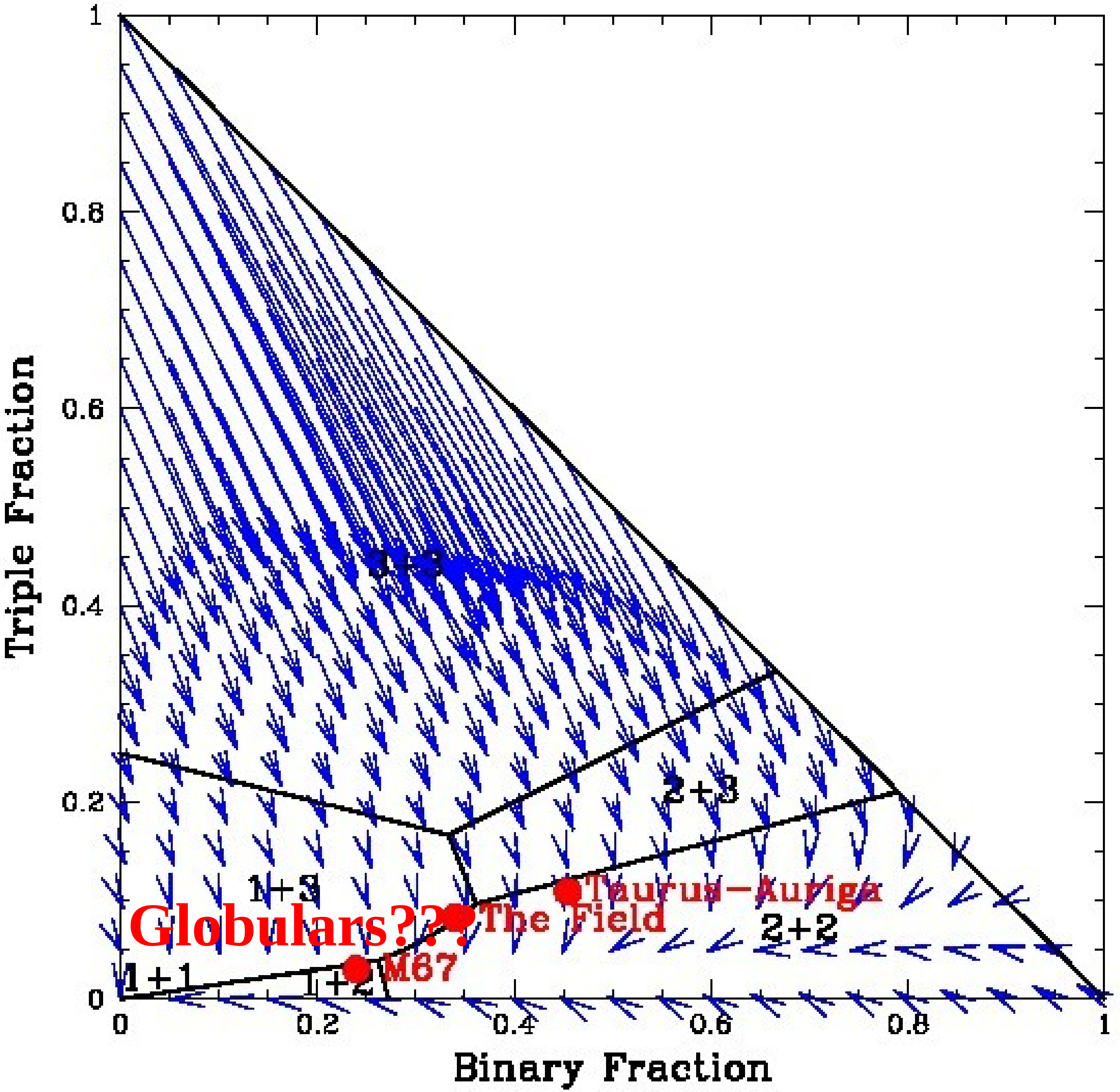
Taurus-Auriga vs M67



The Taurus-Auriga star-forming complex (upper left). **Image credit:** Thomas Preibisch (Munich Observatory).

The open cluster Messier 67. **Image credit:** ThinkingCamera/Flickr via CC.




Triple Fraction
Binary Fraction
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1+1
1+2
1+3
2+2
2+3
3+3
Taurus-Auriga
The Field
M67
Globulars???

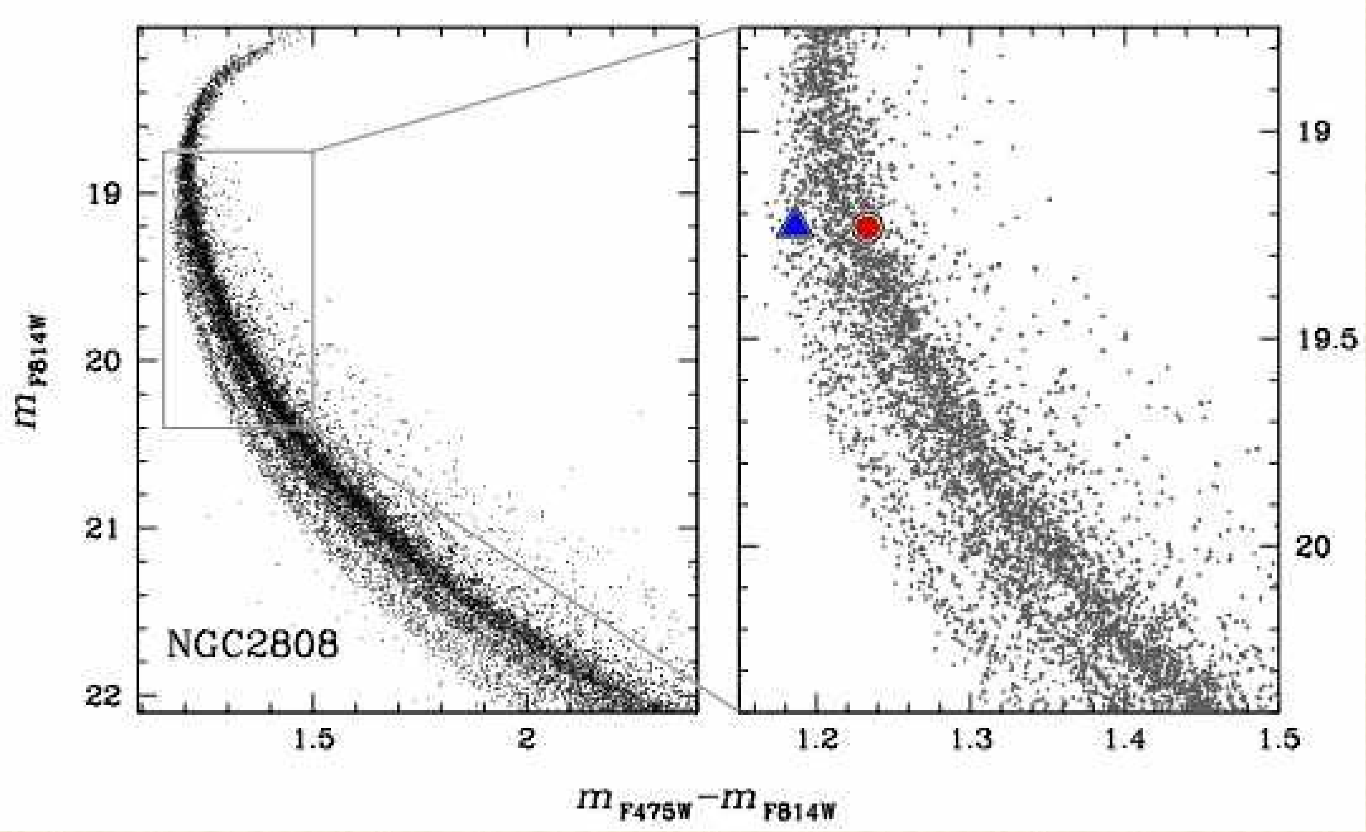
The Message

- N-body and Monte Carlo models for star cluster evolution should include high-order multiplicity (at least triples)
  - Triples provide a key dynamical channel for the formation of stellar exotica – this is ***the*** dominant (dynamical?) channel for the formation of blue stragglers in at least old open clusters (Leigh & Sills 2011; Geller, Hurley & Mathieu 2012)
- This is potentially more than a mere nuisance, since we can ***use*** high-order multiplicity as a dynamical clock

Gas Depletion due to Accretion onto Stellar-mass Black Holes

Multiple Populations in GCs

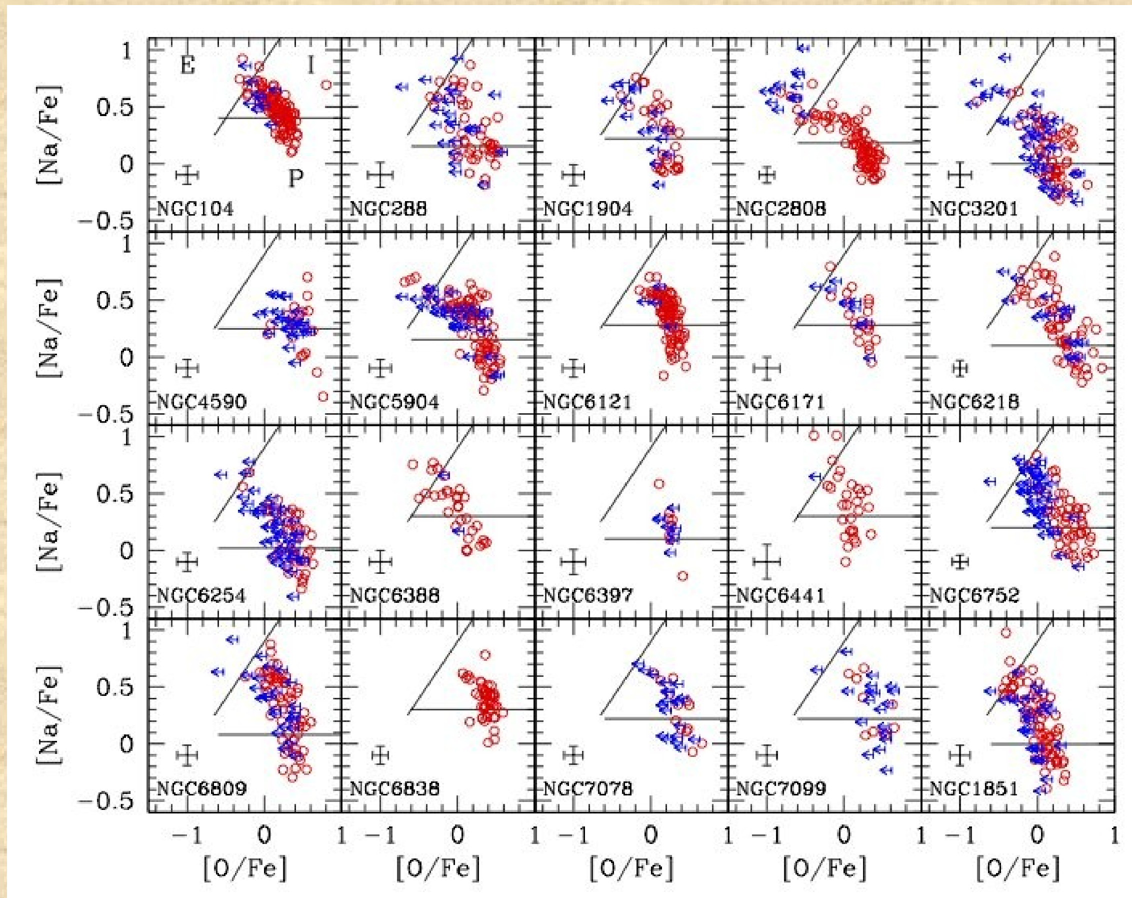
- Distinct evolutionary sequences in CMDs of massive GCs ➔ Multiple episodes of star formation



The triple main-sequence in the Milky Way GC NGC 2808. Figure taken from Figure 1 of Gratton, Carretta & Bragaglia (2012).

Multiple Populations in GCs

- Peculiar chemical abundance anomalies ➔ e.g. no Fe-enrichment in 2nd generation



Na-O anticorrelation observed in massive Milky Way globular clusters. Figure taken from Figure 2 of Gratton, Carretta & Bragaglia 2012.

The Emerging Picture
(e.g. Conroy 2012)

- A first generation of stars is born
- SNeII clear out any remaining gas (?)
- Mass from evolved stars is returned to gas reservoir
- This polluted gas must be diluted by pristine gas (How?)
- A few 100 Myrs later, a second generation is born from this polluted/pristine gas mix
- Star formation ceases after 2nd generation

The Emerging Picture

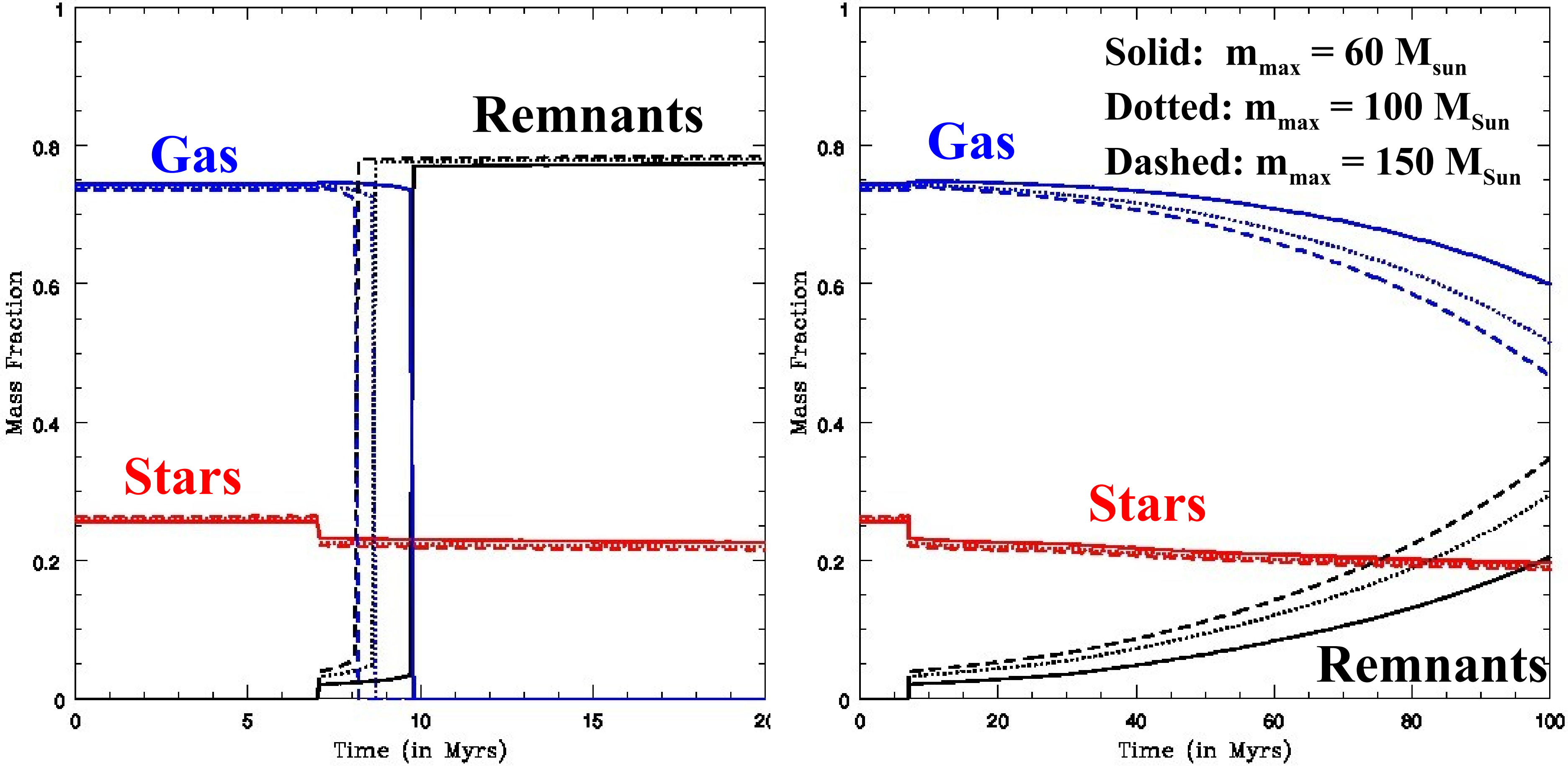
- BUT the escape velocities of clusters with $M > 10^7\ M_{Sun}$ are too high for SNeII to be effective (e.g. Dopita & Smith 1986; Krause et al. 2012)
- Alternative mechanisms for gas depletion other than SNeII and stellar winds?
  - Ram pressure stripping during disk crossings?
  - Pulsar winds?
  - Stellar collisions?

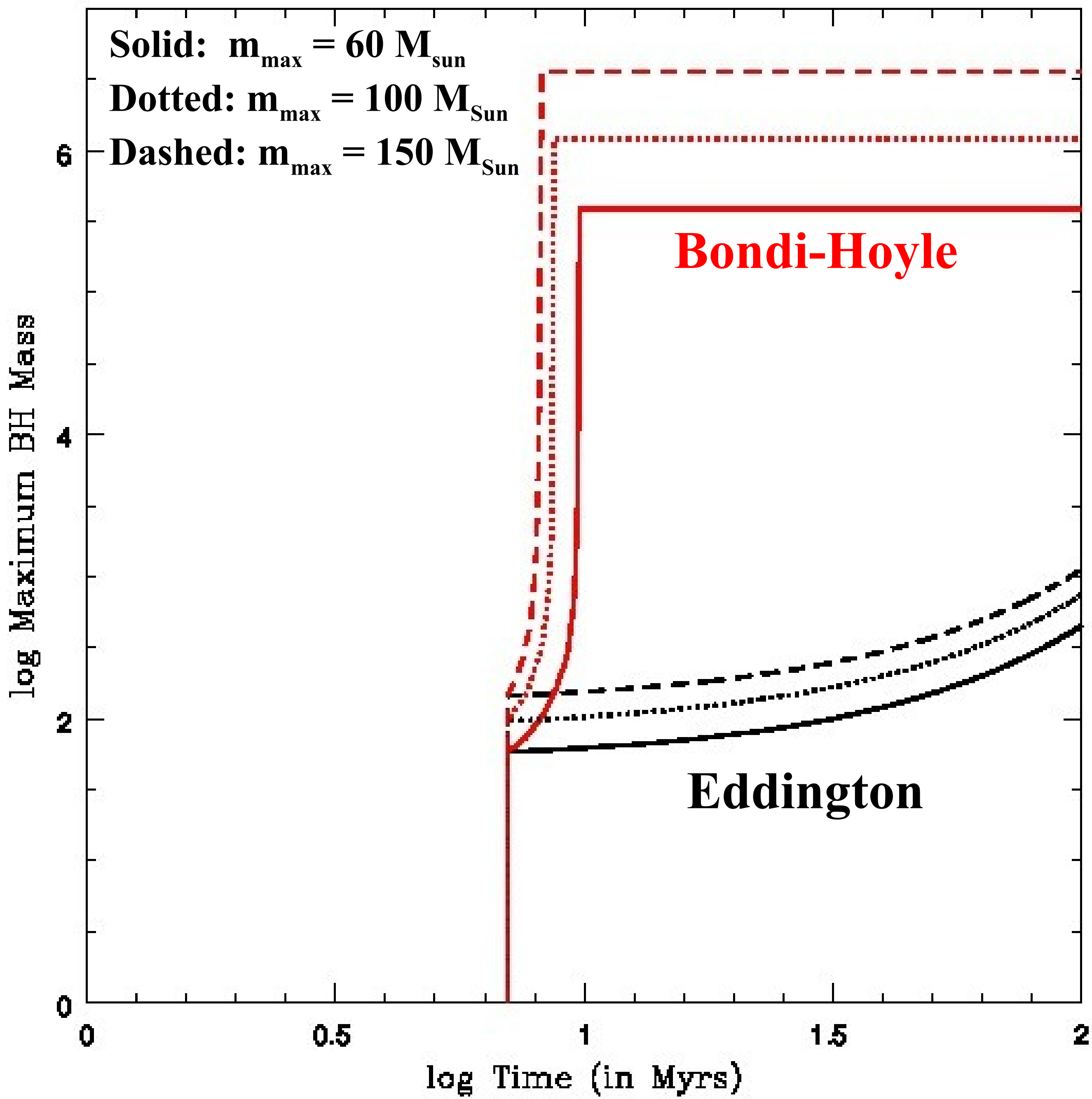
Accretion Onto Black Holes

(see Leigh, Böker, Maccarone & Perets 2012, submitted)

- Construct an analytic model to quantify timescales for gas depletion due to accretion from the ISM onto stellar-mass BHs
- Final timescales are sensitive to:
  - IMF (Salpeter)
  - IMF upper-mass cut-off (60, 100, 150 M_{Sun})
  - Mass-dependency of accretion rate (m^2, m)
  - Gas properties (low and high angular momentum)

Bondi-Hoyle Eddington





Solid: m_{max} = 60 M_{sun}
Dotted: m_{max} = 100 M_{Sun}
Dashed: m_{max} = 150 M_{Sun}
Bondi-Hoyle
Eddington
log Maximum BH Mass
log Time (in Myrs)
0
2
4
6
0
0.5
1
1.5
2

Caveat

- In order for a second stellar generation to be able to form, accretion onto BHs must somehow cease
- Theoretical studies suggest that the timescale for most BHs to be dynamically ejected is typically > 1 Gyr
- BUT accretion increases the BH masses ➔ accelerates the phase of dynamical BH ejections?

The Message

- Accretion onto stellar-mass BHs can significantly deplete the gas reservoir in less than a few 100 Myrs
- This phase of accretion could increase the BH masses sufficiently to prolong the Spitzer instability, and accelerate the phase of dynamical BH ejections
- Accreting BHs could help to account for the absence of Fe-enrichment observed in the second generation

Future Work/Open Questions

- Include "accretion" in N-body models?
  - Slowly increase particle masses over time
  - Reduce particle velocities via momentum conservation
  - Applicable not only to the BH problem considered here, but also to clusters with on-going star formation in general
  - Derive and test an "accretion-modified relaxation time"?
- Reduce computational expense?
  - We need many model realizations to achieve statistical significance

Summary

- For surprisingly low numbers of high-order multiples, they can be as dynamically-relevant as binaries
- Observationally-measured binary and triple fractions offer a new tool for the relative dynamical age-dating of star clusters
- Can we use this tool to constrain the origins of stars in the Galactic Field?
- These techniques are also applicable to asteroids in the solar system, and molecular species in gas clouds

Historical Perspective

- Three-body problem first considered by Newton in his Principia in 1687
- Later tackled by several big names, including Euler, Lagrange, Jacobi, Poincaré and Hill
- These heroic souls sought an exact solution that describes the motion of three celestial bodies under their mutual gravitational attraction
- Sadly, they perished “sans solution”, and little progress was made for centuries

The Revolution

- A few years after Poincaré, a new approach began: integration of orbits step-by-step
- Computers revolutionized the three-body problem using this approach
- Very few studies have considered more than 3 bodies
    - Long integration times to run simulations to completion
    - Extensive parameter space to explore
    - Number of possible outcomes increases steeply with N

Astrophysical Applications

- Galactic centre
  - Formation of stellar exotica (e.g. LMXBs, S stars, etc.)
  - SMBH growth via runaway collisions?
  - Hypervelocity stars
  - Tidal flares
- Young star-forming clusters
  - Runaway O/B stars
  - Initial clump infall
  - Stellar mergers and the IMF
- Dense star clusters
  - Formation of stellar exotica (e.g. LMXBs, CVs, MSPs, blue stragglers)
  - Implications for cluster evolution
  - Escape of stars into field (GAIA?)

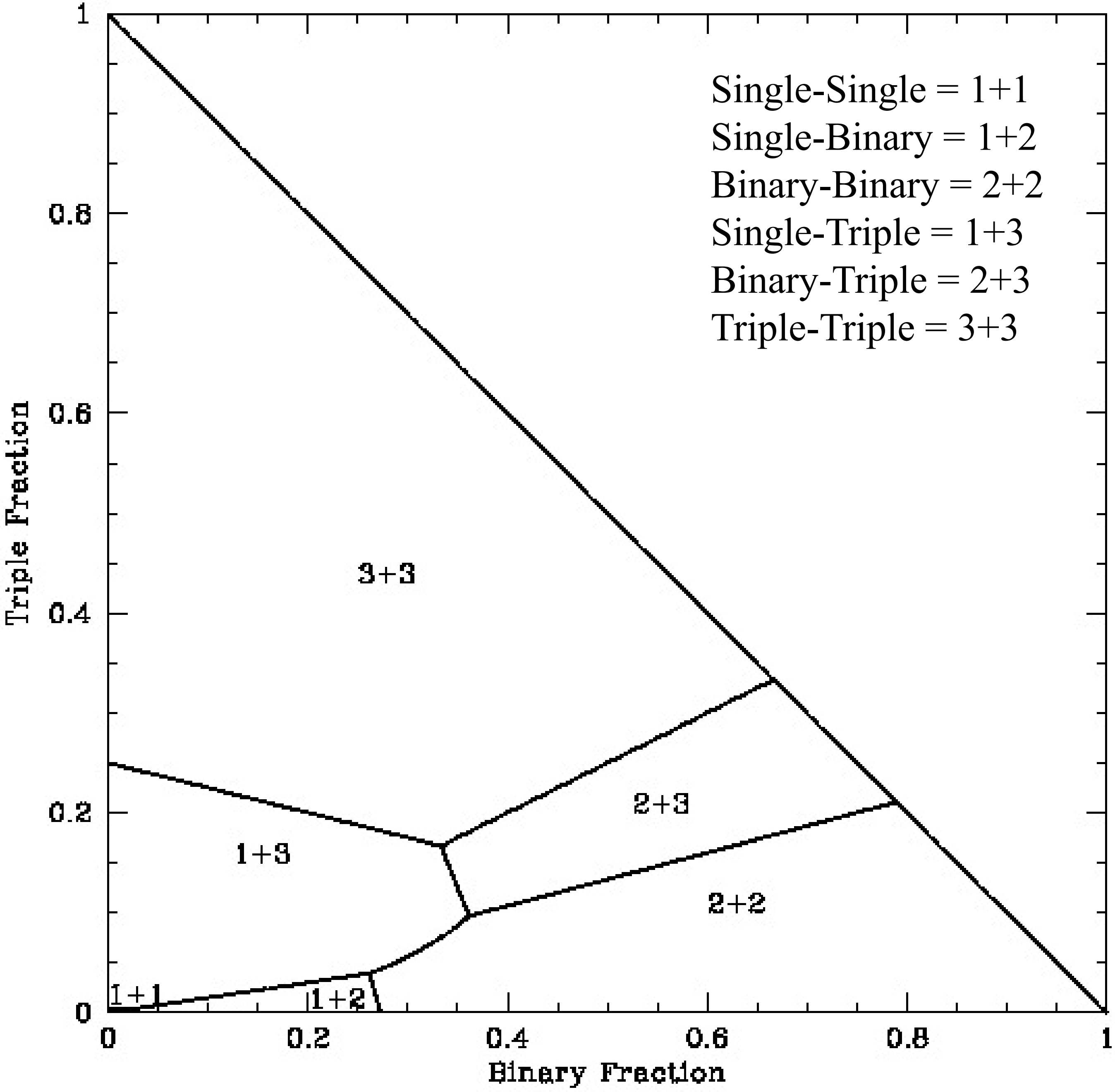
Motivation for the Present Study

- Triples have been identified in significant numbers in several moderately-dense old open clusters (e.g. M67, NGC 6791, NGC 188, etc.)

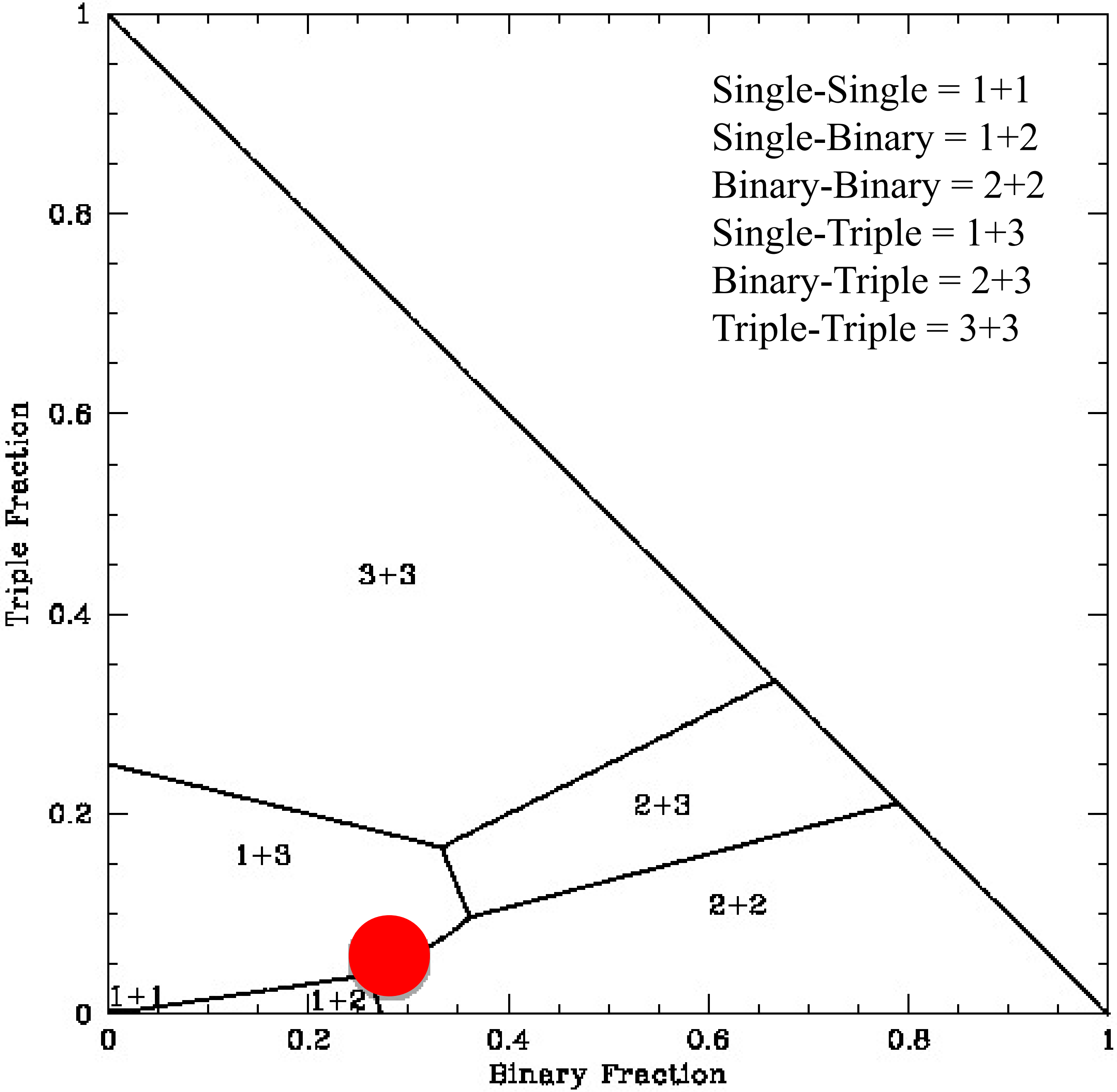
$$f_{bin}/f_{trip} \sim 5$$

- Triples undergo encounters with other objects more than either single or binary stars

  ➔ This is likely the dominant dynamical mechanism for stellar collisions to occur, *at least* in these clusters



Single-Single = 1+1
Single-Binary = 1+2
Binary-Binary = 2+2
Single-Triple = 1+3
Binary-Triple = 2+3
Triple-Triple = 3+3
3+3
1+3
2+3
2+2
1+1
1+2
Triple Fraction
Binary Fraction
0
0.2
0.4
0.6
0.8
1


Single-Single = 1+1
Single-Binary = 1+2
Binary-Binary = 2+2
Single-Triple = 1+3
Binary-Triple = 2+3
Triple-Triple = 3+3
3+3
2+3
1+3
2+2
1+1
1+2
Triple Fraction
Binary Fraction
0
0.2
0.4
0.6
0.8
1

The Problem at Hand

Question: How does the probability of a collision occurring depend on the number of objects involved in the interaction?

What is the "N-dependence"?

Method

1. Define a *normalization* to compare between 2+2, 1+3, 2+3, 3+3 encounters
2. Choose the initial conditions
3. Perform $> 10^6$ numerical scattering experiments for 1+2, 2+2, 1+3, 2+3, 3+3 encounters
4. Isolate the N-dependence of the collision probability

1. Normalization

- What are the initial parameters of the encounters that determine the outcomes?
- Our normalization must remove these dependences in order to isolate the N-dependence of the collision probability
- Fix both the **total energy** and **total angular momentum** when comparing different encounter types

2. Initial Conditions

Encounter Type	Run 1 (in AU)	Run 2 (in AU)	Run 3 (in AU)
1+ 2	10.0	5.0	15.0
2+ 2	1.0; 10.0	0.5; 5.0	2.0; 15.0
1+ 3	(1.0, 10.0)	(0.5, 5.0)	(2.0, 15.0)
2+ 3	10.0; (1.0, 10.0)	5.0; (0.5, 5.0)	15.0; (2.0, 15.0)
3+ 3	(1.0, 10.0); (1.0, 10.0)	(0.5, 5.0); (0.5, 5.0)	(2.0, 15.0); (2.0, 15.0)

* Each entry provides the semi-major axis in AU of all orbits involved in the encounter.

** The semi-major axes for triples are given in the form: (a_{in}, a_{out}).

***All stars are 1 M_{Sun} and all orbits are circular.

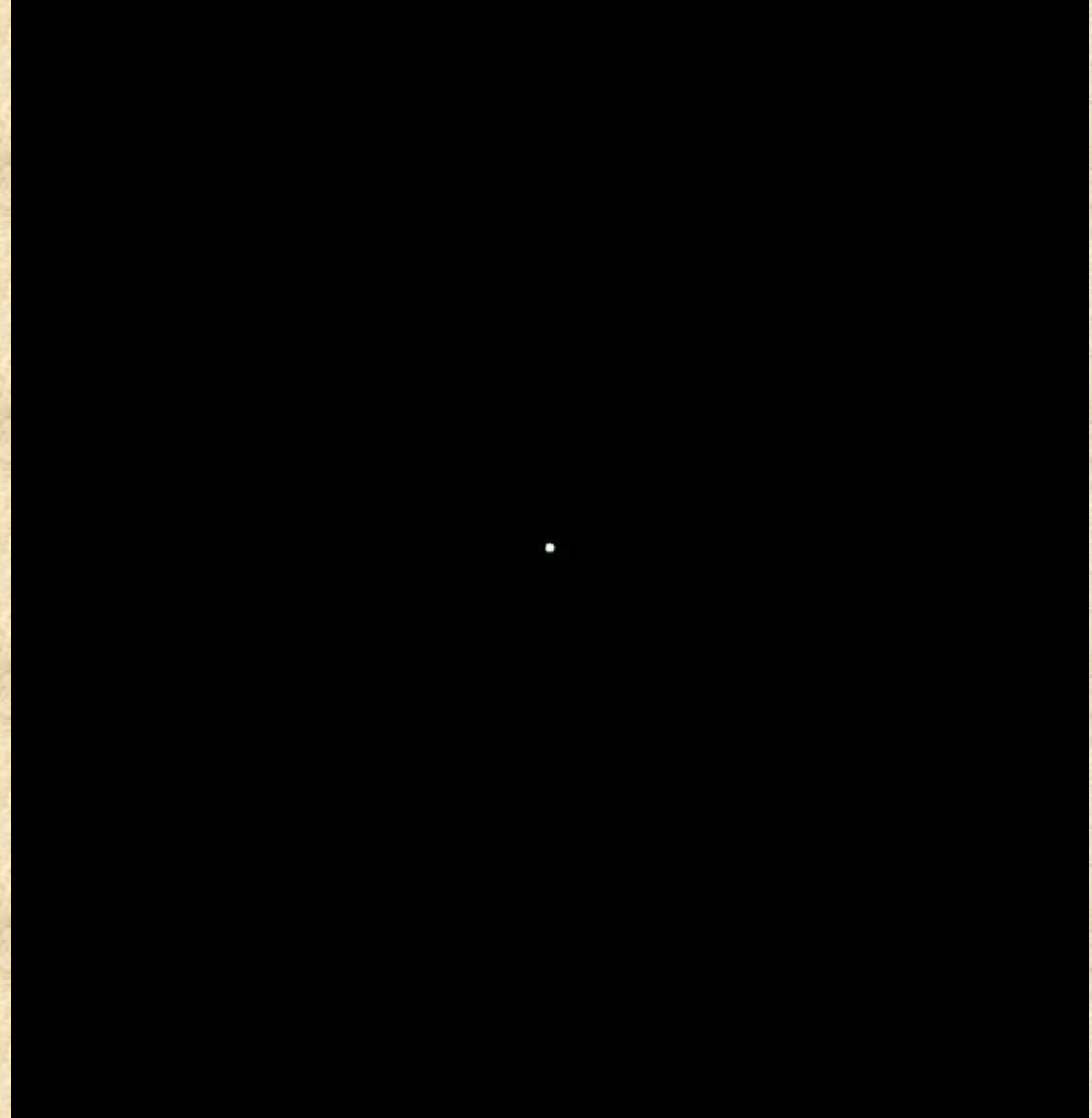
3. Experiments

- Upgraded the FEWBODY numerical scattering code to simulate encounters involving triples

  ➔ http://fewbody.sourceforge.net

- Low angular momentum regime
- For several sets of different conditions, we performed 800,000 simulations for every encounter type

Collision Probability = N /N



Movie credit: Aaron Geller (Northwestern University)

4. N-Dependence

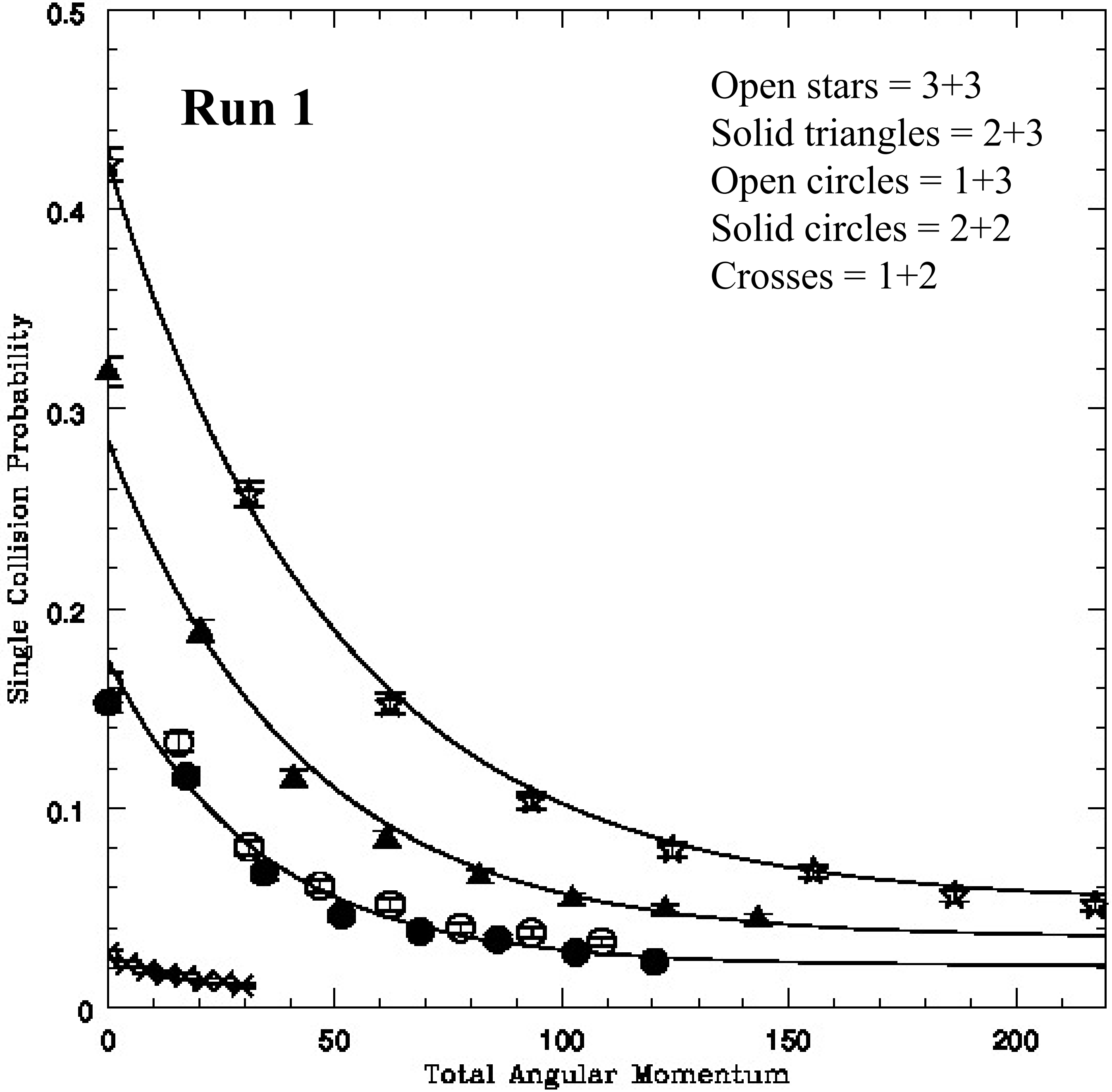
- Choose a general functional form for the collision probability

$$P_{coll}(q) = \alpha N^{\beta} C(N,L) q/a_0$$

where

$$C(N,L) = \exp(-\delta N/L) + \gamma$$

- Fit this function to the results of our scattering experiments, and obtain the best-fit parameters


Run 1
Open stars = 3+3
Solid triangles = 2+3
Open circles = 1+3
Solid circles = 2+2
Crosses = 1+2
Single Collision Probability
Total Angular Momentum
0
0.1
0.2
0.3
0.4
0.5
0
50
100
150
200

Results

Parameter	Run 1	Run 2	Run 3
α	0.16 ± 0.02	1.56 ± 0.24	2.15 ± 0.52
β	2.19 ± 0.09	1.93 ± 0.09	2.33 ± 0.15
δ	-0.12 ± 0.01	-0.09 ± 0.01	-0.17 ± 0.02
γ	0.14 ± 0.01	0.14 ± 0.01	0.23 ± 0.03

Interpretation?

- Consider two possibilities:

1. $N_{cc} \sim N^2$ with N_{cross} = constant
2. $N_{cross} \sim N^2$ with N_{cc} = constant

Interpretation?

- Consider two possibilities:

    1. $N_{cc} \sim N^2$ with N_{cross} = constant
    2. $N_{cross} \sim N^2$ with N_{cc} = constant

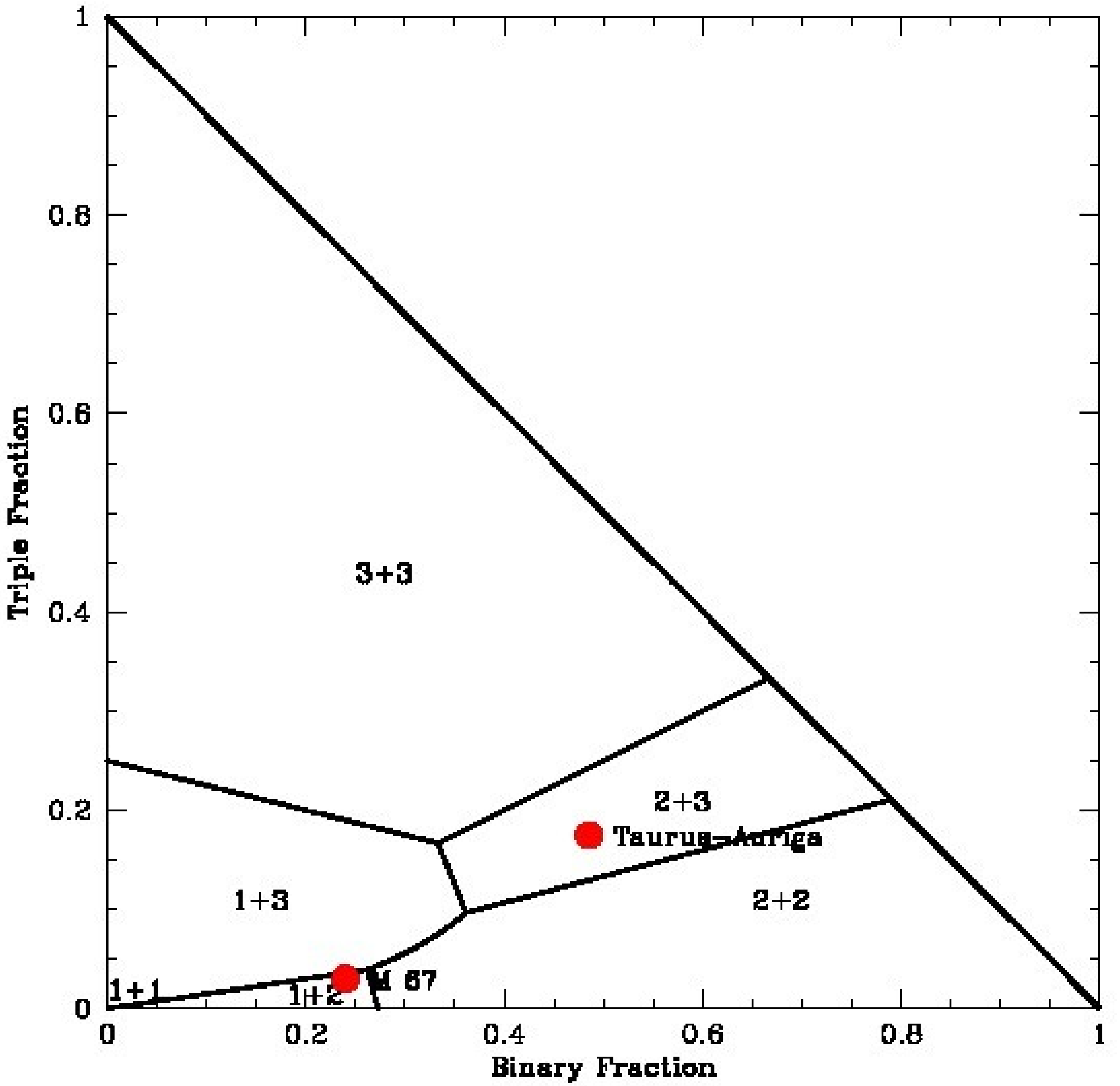
Interpretation?

- Consider two possibilities:

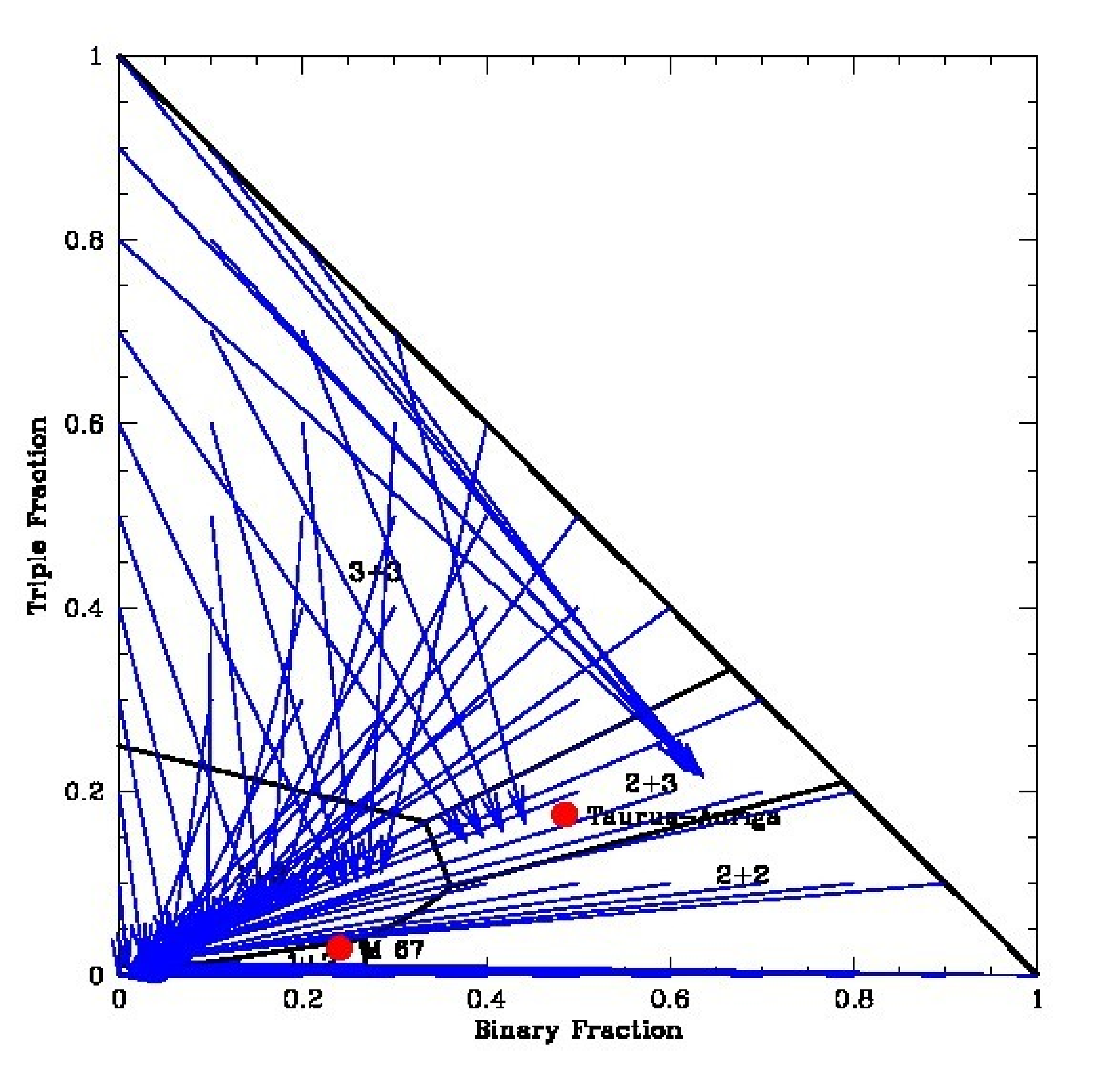
  1. $N_{cc} \sim N^2$ with N_{cross} = constant
  2. $N_{cross} \sim N^2$ with N_{cc} = constant

- In the limit of very large N, we have $\boldsymbol{\Gamma_{coll} \sim N^2}$, where Γ_{coll} is the collision rate derived from the **mean-free path approximation**

  ➔ Coincidence?



Are these multiplicity fractions primordial, or have they been altered by the cluster dynamics?



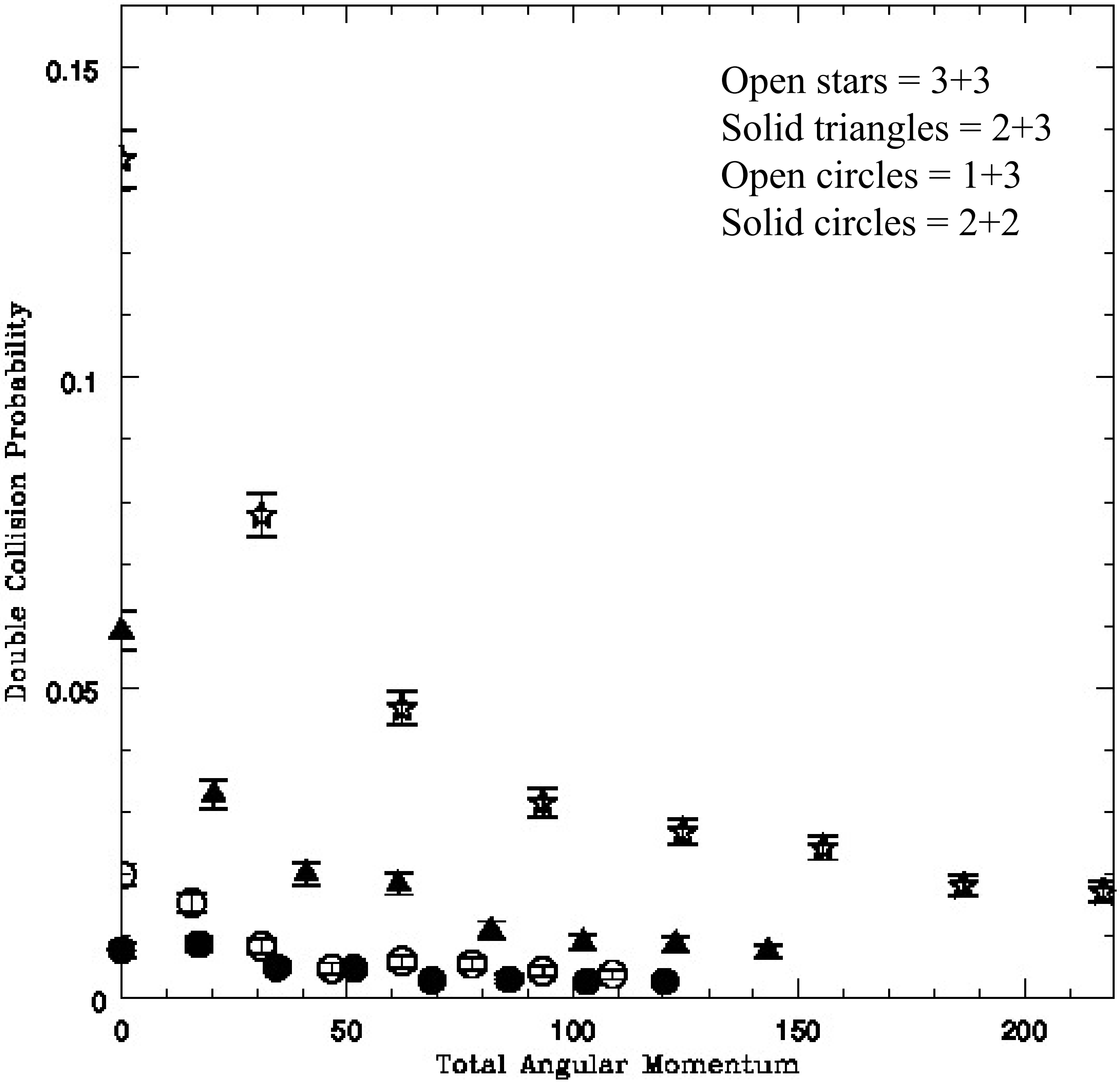
Evolution in the binary fraction-triple fraction plane due to dynamical interactions.

Summary

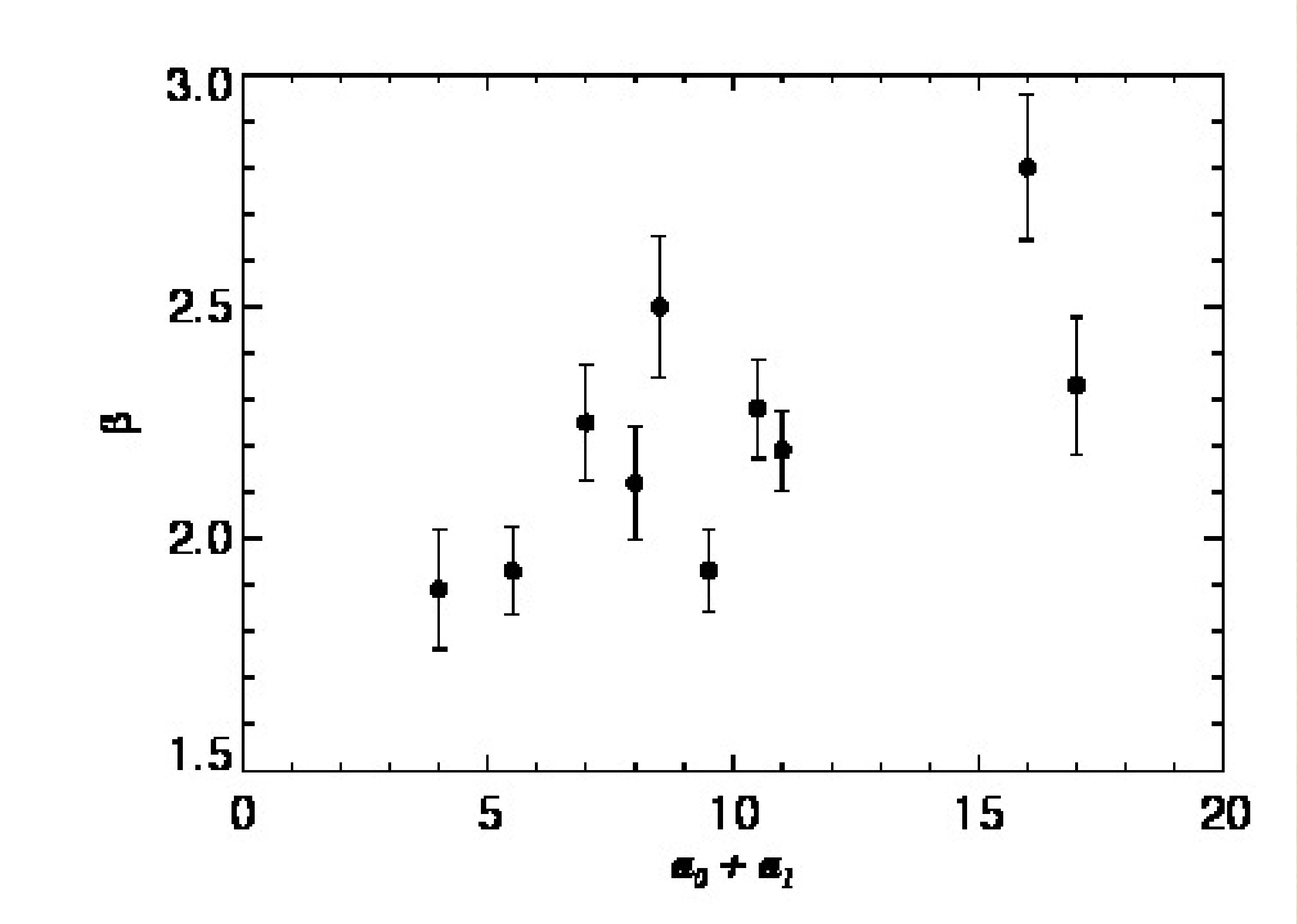
- Presented a new formalism to quantify the N-dependence of the collision probability
- Short encounter times + High collision probabilities = Encounters with triples should be the main channel for collisions in at least old open clusters
- By extension, triples could be important catalysts for the formation of stellar exotica (blue stragglers, LMXBs, MSPs, etc.)
- Shown how the observed binary and triple fractions can be used to constrain the *relative* dynamical ages of clusters

Summary

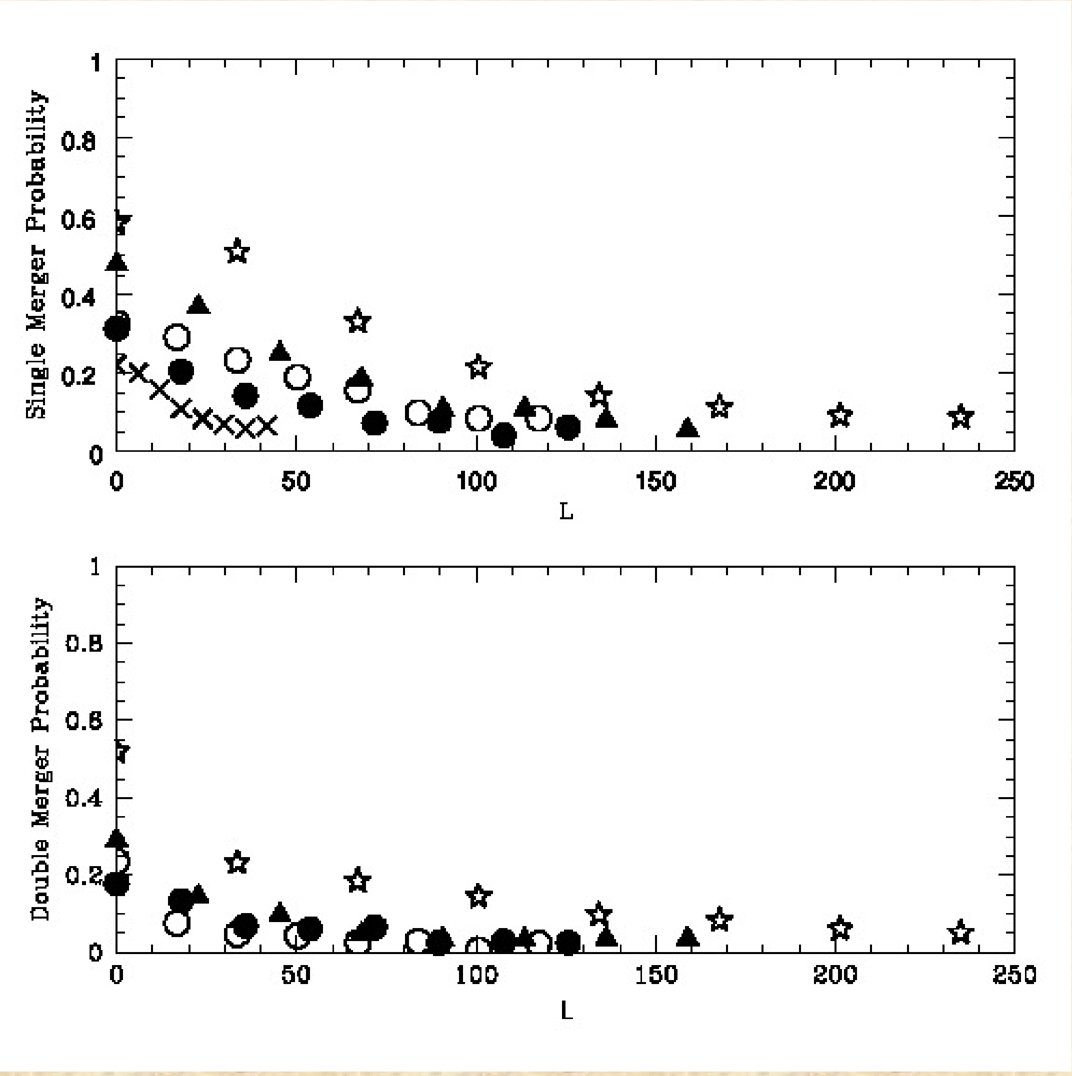
- Presented a new formalism to quantify the N-dependence of the collision probability
- Applied it to the results of $>10^6$ numerical scattering experiments of encounters involving up to 6 objects
- Short encounter times + High collision probabilities = Encounters with triples could be the main channel for collisions in at least old open clusters
- Collision probability scales as N^2 ➔ Connection to the mean free path approximation?


Open stars = 3+3
Solid triangles = 2+3
Open circles = 1+3
Solid circles = 2+2
Double Collision Probability
Total Angular Momentum
0
0.05
0.1
0.15
0
50
100
150
200

Results



2. Find Collision Probability



Crosses = 1+2
Open stars = 3+3
Solid triangles = 2+3
Open circles = 1+3
Solid circles = 2+2

Run 2

3. Create a Simple Model

- Assume that system evolves via a succession of ejections until an escape takes place
    - At each ejection, a temporary binary is formed ➔ close two-body encounters at pericentre, $q = a(1 - e)$, of every orbit
    - a does not vary greatly between ejections ($a \sim a_0/2$), but e does (over the entire range of the distribution $f(e)\,de = 2e\,de$)
- Probability of close approach $P(q)$ within distance q per ejection is

  ➔ $1 - e^2 = (1 - e)(1 + e) = (q/a)(1 + e)$, since $q = a(1 - e)$, and $1 - e^2$ is probability that eccentricity is $> e$

  $\sim 2q/a$, for $e \sim 1$

  ➔ Total probability $\sim 120q/a$, from $f(e) = 2e\,de$, system survives for ~ 30 crossing times, and assuming very small q/a
- On average, one ejection per crossing time, and two close approaches (Szebehely & Peters 1967)

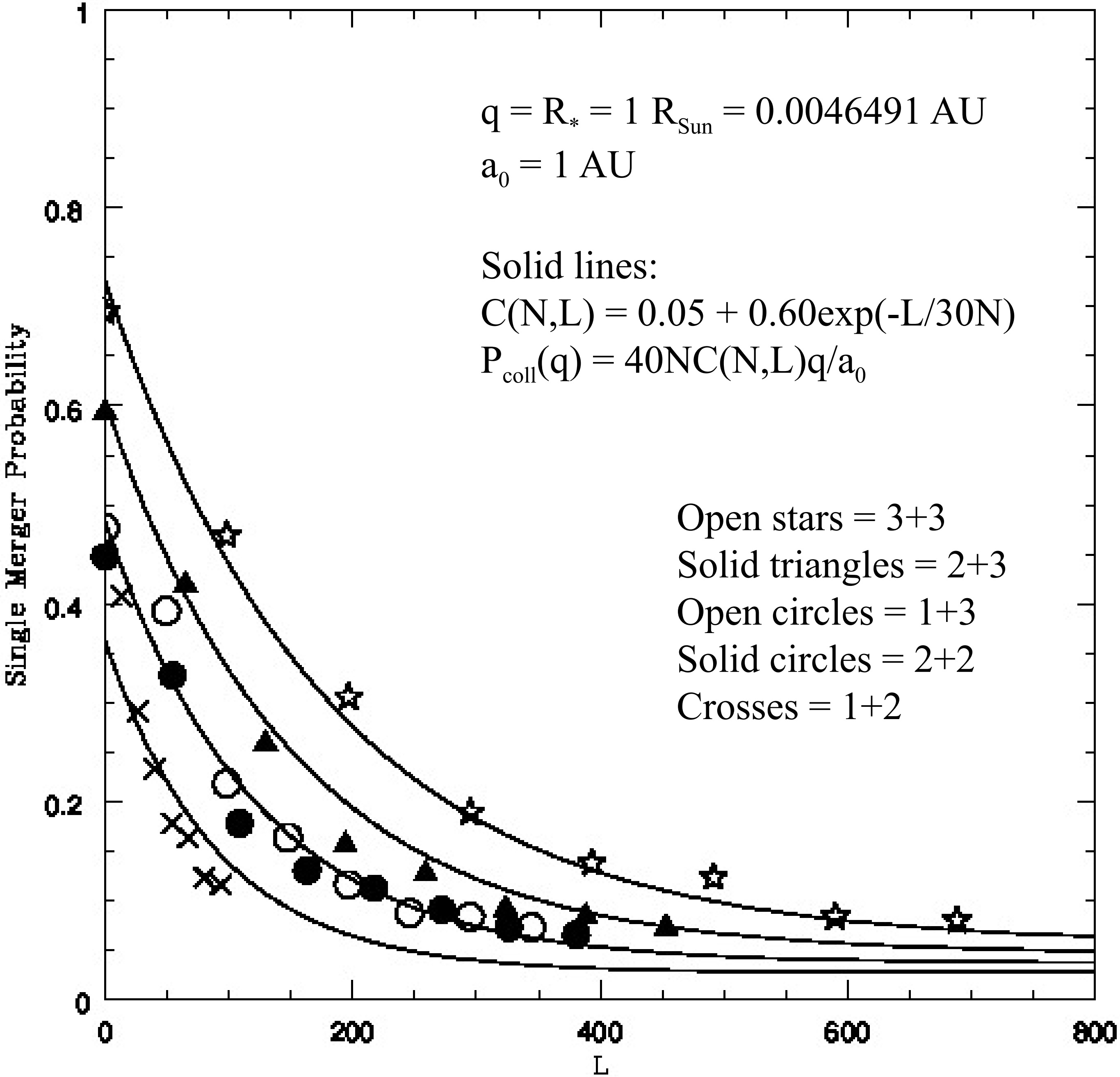
3. Create a Simple Model

- BUT $P(q)$ should depend on the total angular momentum L (from numerical scattering experiments, since the interaction lifetime depends on L)

  ➔ $P(q) \sim 240C(L)q/a_0$,

  where $C(L) \sim 1 + 7.5L^2$ (from numerical scattering experiments; Saslaw et al. 1974)

- Take $q \sim R_*$, and $P(q)$ = probability of a collision occurring
- ***Can this simple model be extended to accurately describe higher N interactions?***



$q = R_* = 1\ R_{Sun} = 0.0046491$ AU
$a_0 = 1$ AU
Solid lines:
$C(N,L) = 0.05 + 0.60\exp(-L/30N)$
$P_{coll}(q) = 40NC(N,L)q/a_0$
Open stars = 3+3
Solid triangles = 2+3
Open circles = 1+3
Solid circles = 2+2
Crosses = 1+2
Single Merger Probability
L
0
0.2
0.4
0.6
0.8
1
0
200
400
600
800

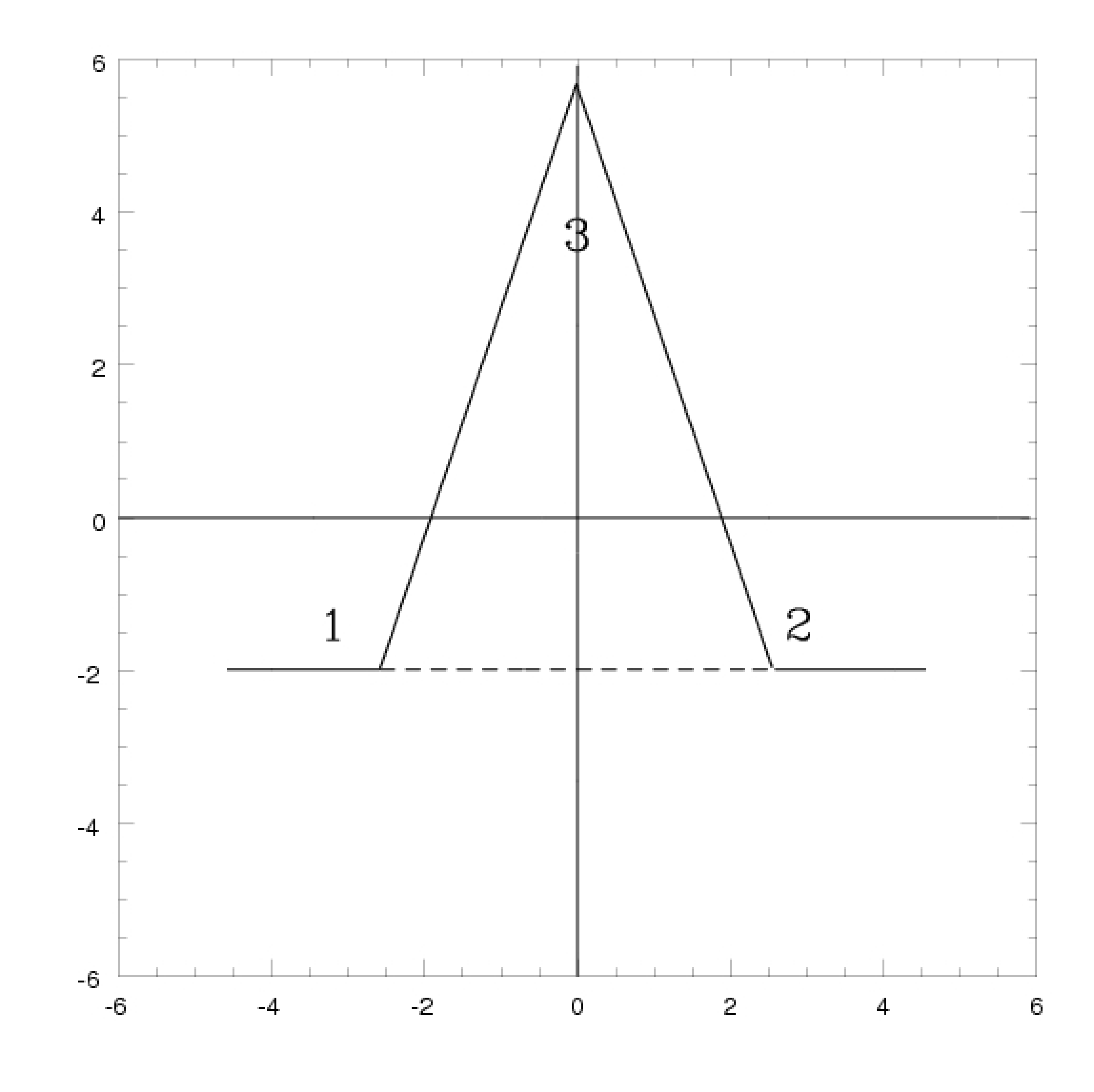
Do these fits hold up for different initial conditions?

- Tried making the same plots for Runs 2 and 3
- Simple model doesn’t hold up so well, at least not without further adjustments…

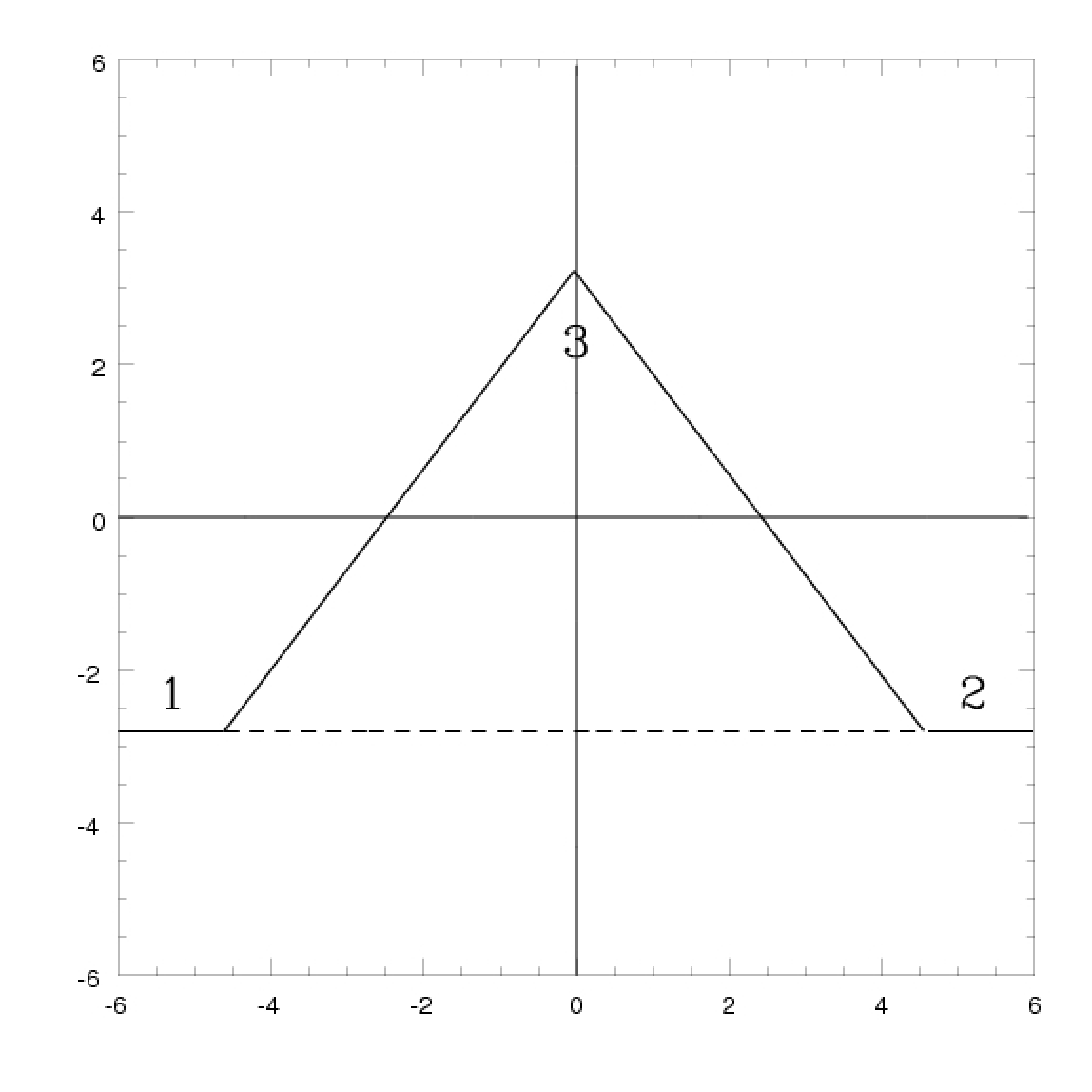
4. Describe results of encounters in a straight-forward way

- Consider an encounter involving 3 stars
- Let $E_i = T_i + V_i$ be the total energy of star i with respect to the center of mass of the system
- The total energy is then $E_{tot} = \sum E_i$
- Consider a triangle for which we define the angles as $\theta_i = -180°(E_i/E_{tot})$

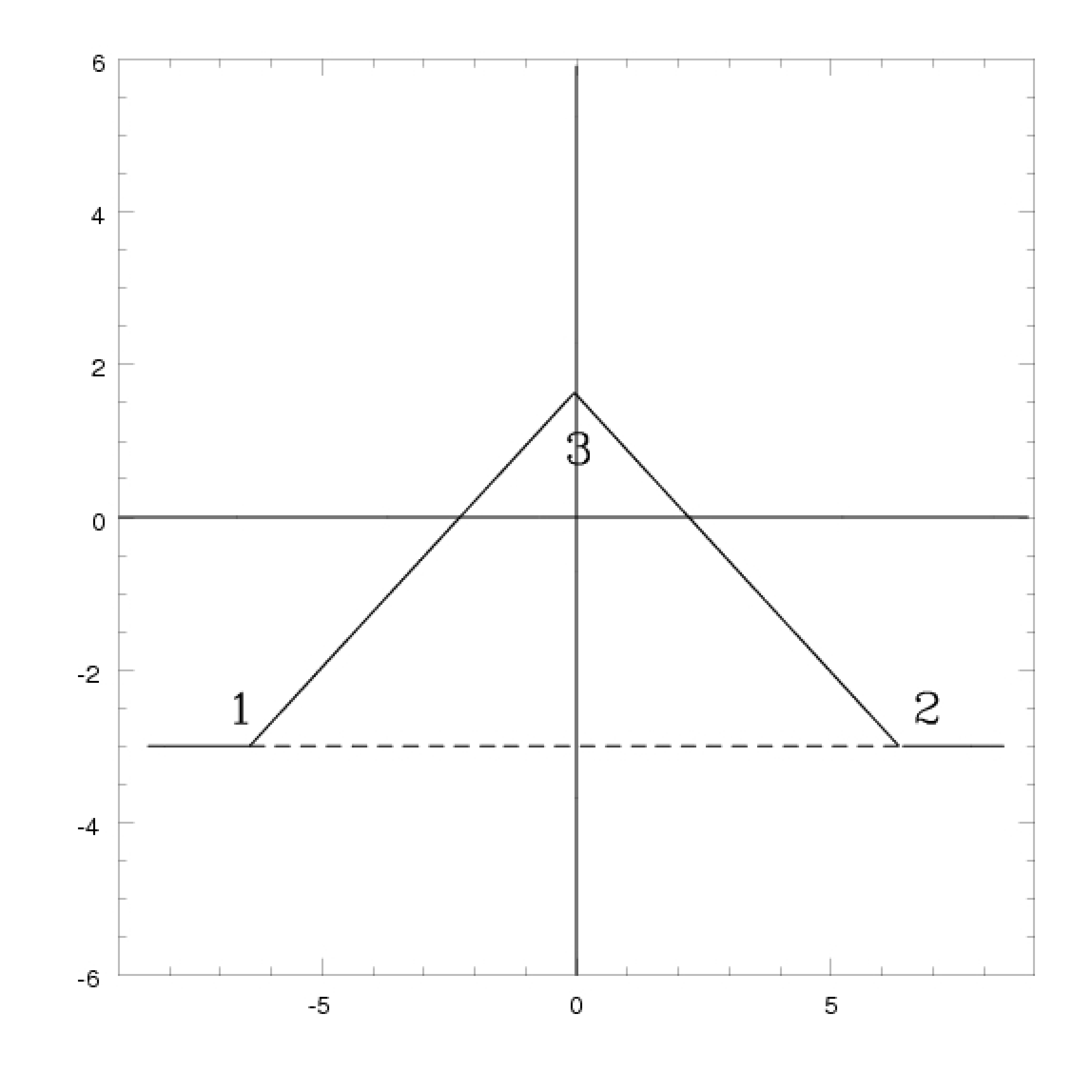
➔ Permutations of the triangle provide a visual representation of the evolution of the 3-body interaction in *energy space*



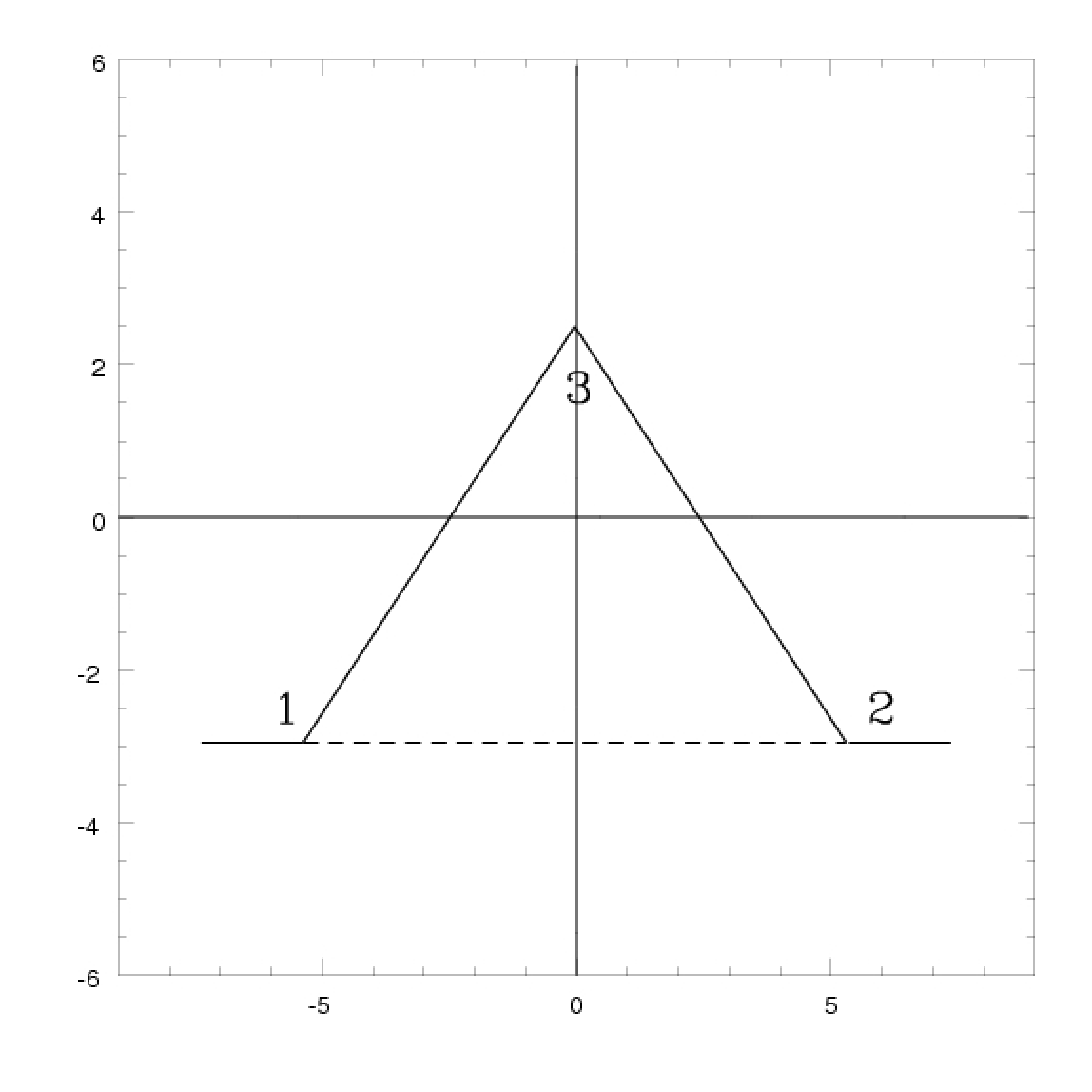
Schematic diagram showing the **initial** distribution of energies for a typical **hardening** encounter between a single star and a binary. Stars 1 and 2 correspond to the components of the binary, whereas star 3 is the interloping single star.



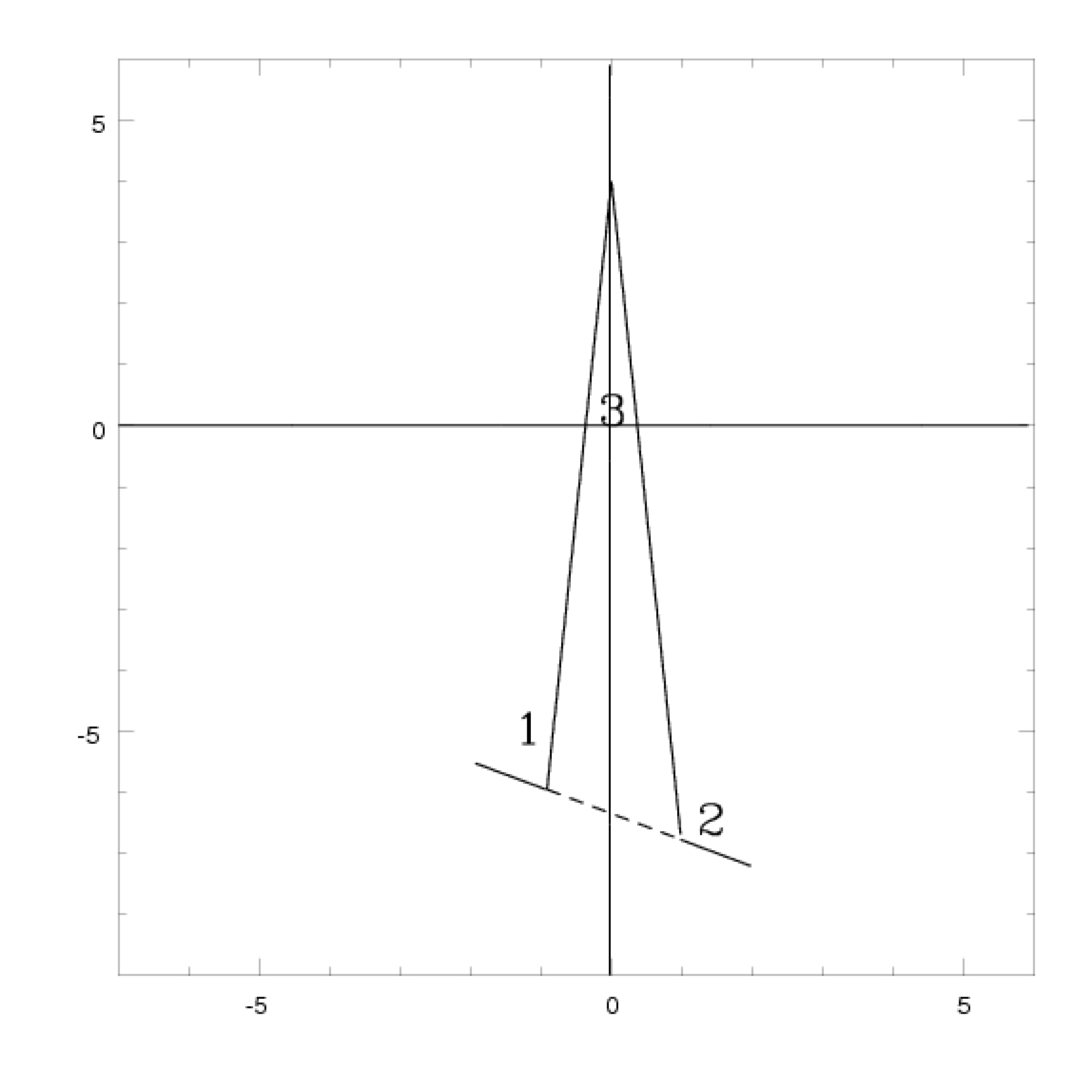
Schematic diagram showing the **final** distribution of energies for a typical **hardening** encounter between a single star and a binary. Stars 1 and 2 correspond to the components of the binary, whereas star 3 is the interloping single star.



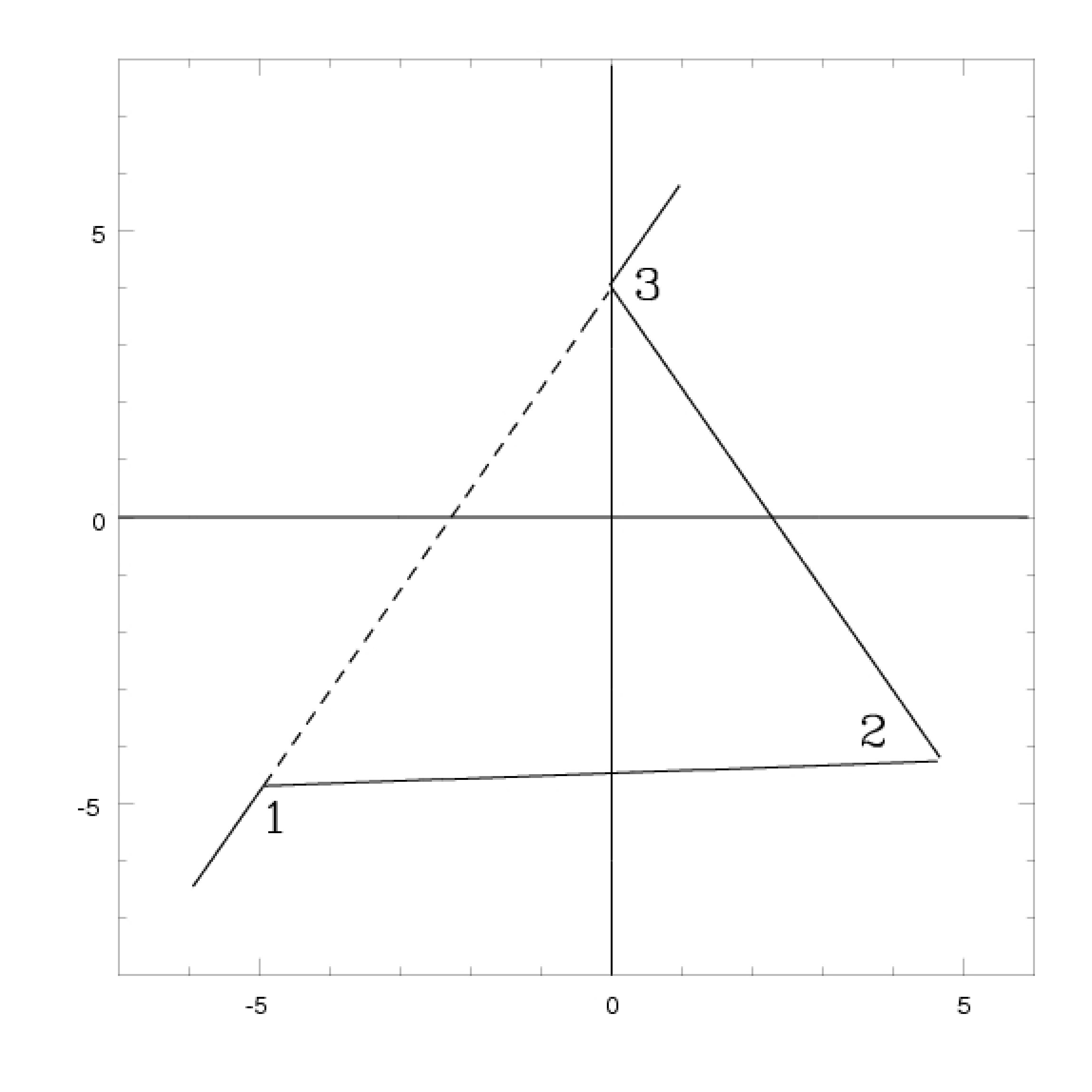
Schematic diagram showing the **initial** distribution of energies for a typical **softening** encounter between a single star and a binary. Stars 1 and 2 correspond to the components of the binary, whereas star 3 is the interloping single star.



Schematic diagram showing the **final** distribution of energies for a typical **softening** encounter between a single star and a binary. Stars 1 and 2 correspond to the components of the binary, whereas star 3 is the interloping single star.



Schematic diagram showing the **initial** distribution of energies for a typical **exchange** encounter between a single star and a binary. Stars 1 and 2 correspond to the components of the binary, whereas star 3 is the interloping single star.



Schematic diagram showing the **final** distribution of energies for a typical **exchange** encounter between a single star and a binary. Stars 1 and 2 correspond to the components of the binary, whereas star 3 is the interloping single star.

Historical Perspective

- **Problem:** How can we find an exact solution that describes the motion of three celestial bodies under their mutual gravitational attraction?

- Tackled by all the big names, including Newton, Euler, Lagrange, Jacobi, Poincare and Hill

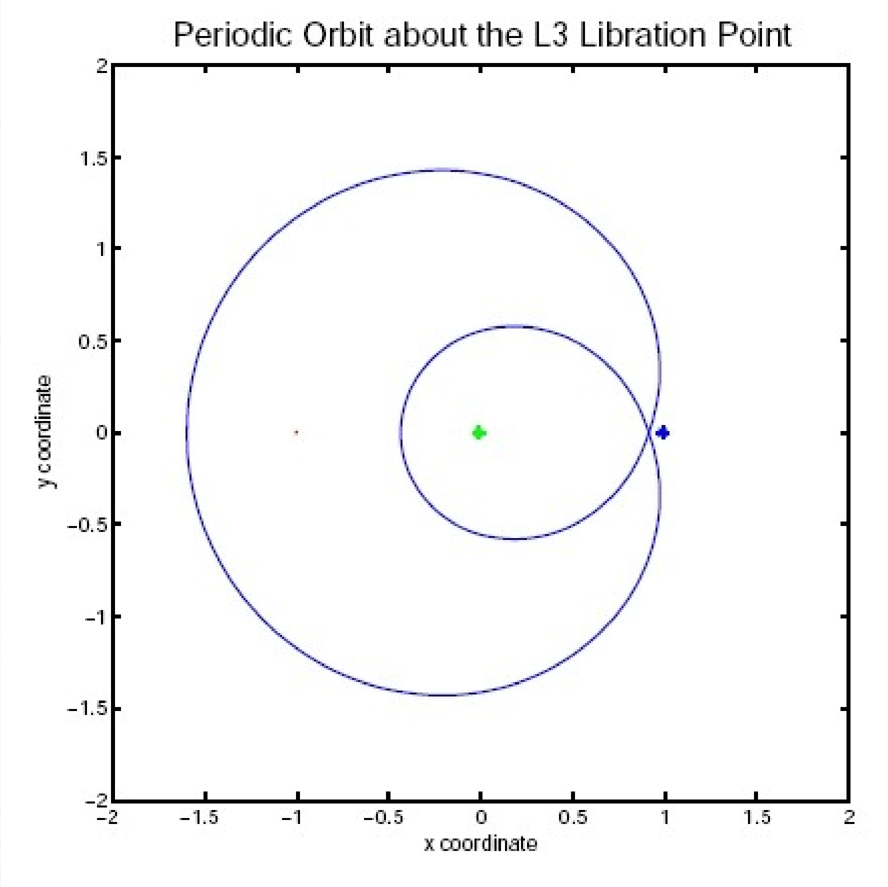
The Three-Body Problem: Special Cases

- **Basic Idea:** make assumptions that reduce the number of unknown variables in the equations of motion

  $\Rightarrow$ reduces the problem to a solvable system of equations with an appropriate number of unknown variables

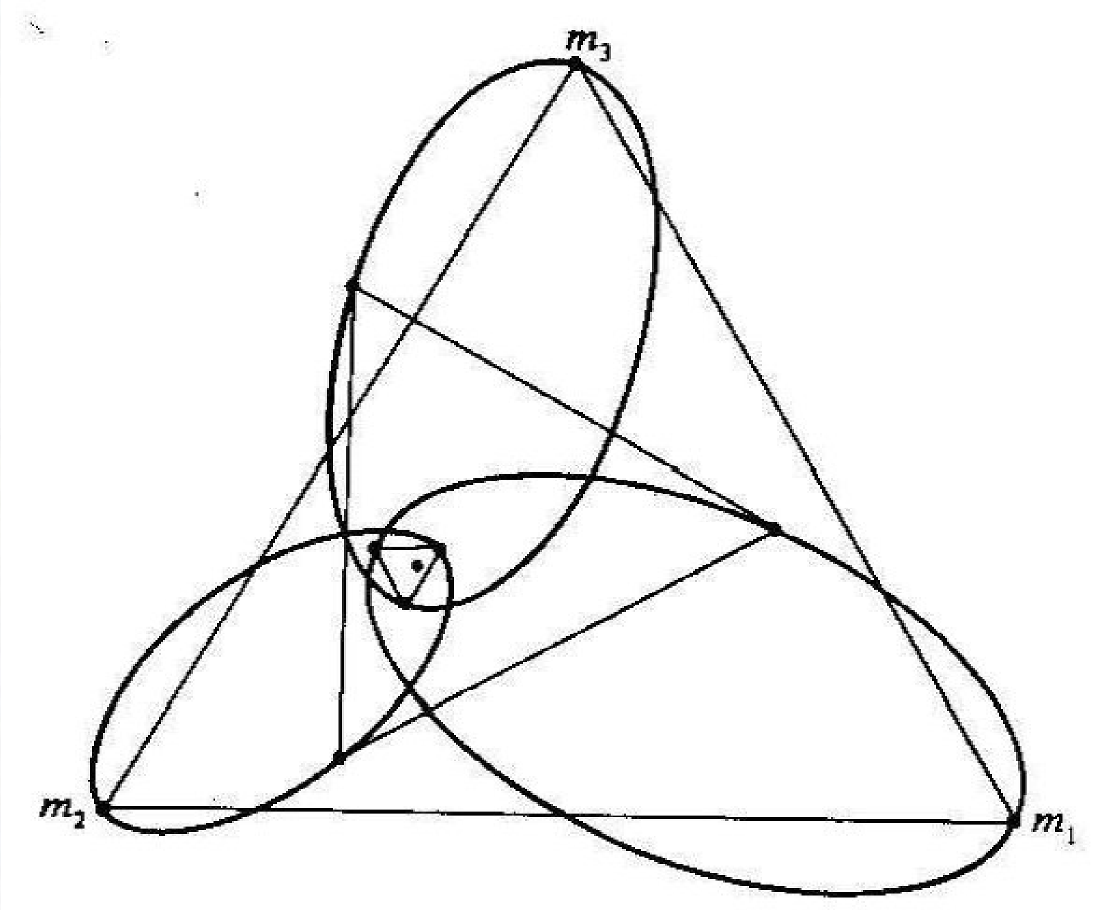
The Planar Restricted Circular Three-Body Problem

- **Restricted** ⇒ mass of third body is very small
- **Circular** ⇒ primaries move in circular orbits
- **Planar** ⇒ third body moves in the same plane as the primaries



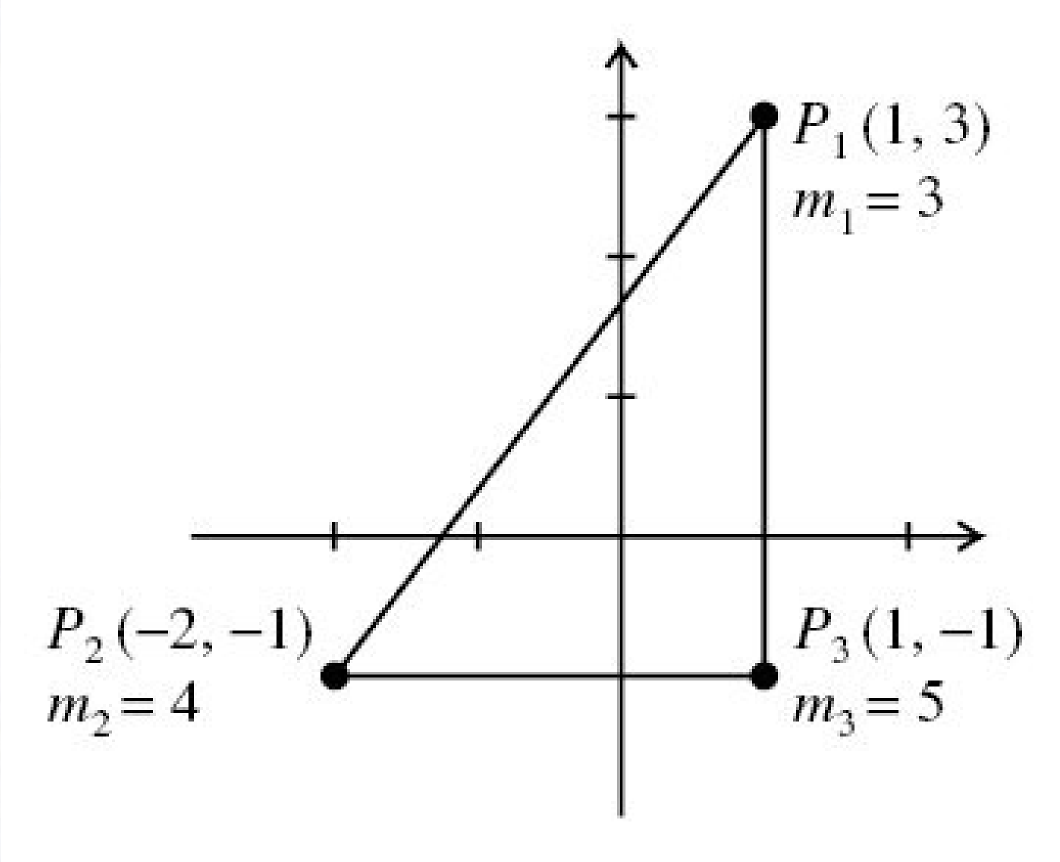
⇒ An exact solution for this problem can be found, and the equations of motion can be solved analytically.

The Lagrangian Equilateral Triangle

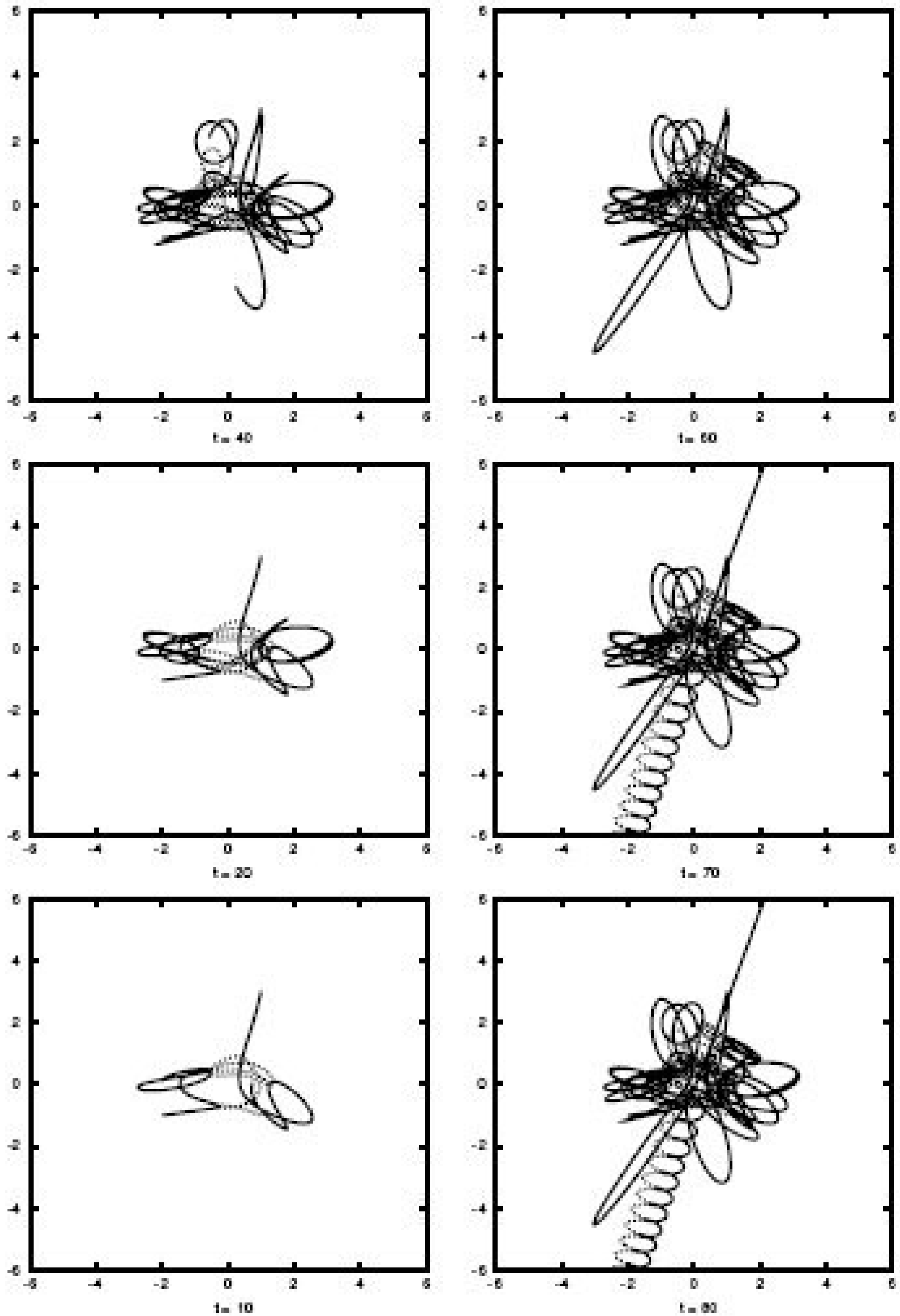


Lagrangian equilateral triangle and possible orbits.

Pythagorean Three-Body Problem



Three bodies are initially placed at the vertices of a Pythagorean right triangle. The masses of the bodies are 3, 4 and 5 (in whatever units), and the length of the sides correspond to the respective masses.

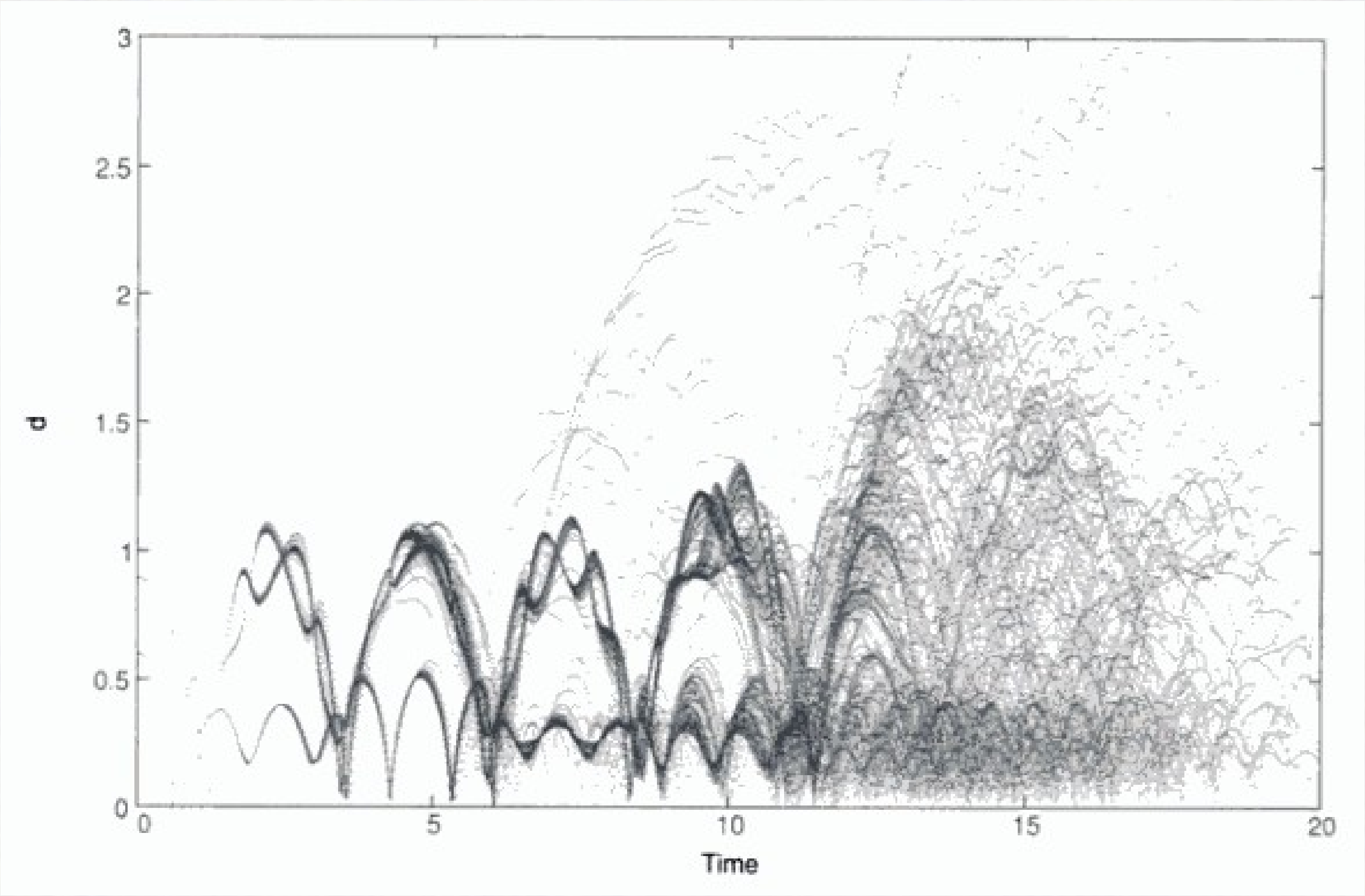


Without simplifying assumptions, the evolution of three-body interactions are typically chaotic…

Simulated Three-Body Encounter

- Three-body exchange interaction calculated using the FEWBODY scattering code; courtesy of Dr. Aaron Geller (Northwestern University)

Chaos in the Three-Body Problem



The time evolution of a three-body system with small variations in in the initial position of one of the bodies. The y-axis shows the distances of the three bodies from the center of mass, while the x-axis is time in units of crossing time.

The General Three-Body Problem

- **Problem:** How can we predict the outcomes of three-body interactions given a set of initial conditions?
- **Question:** Although an exact solution seems unattainable, can we identify general trends without drastically restricting the applicable parameter space?

Heggie's Law

- Consider a star cluster composed of binary and single stars

$$\frac{1}{2} m\sigma^2 = Gm^2/2a$$

Typical Kinetic Energy of Single Stars	=	Orbital Energy of Typical Binary

$\Rightarrow$ Binaries with $\mathbf{a < Gm/\sigma^2}$ get "**harder**", binaries with $\mathbf{a > Gm/\sigma^2}$ get "**softer**"

Simulated Three-Body Encounter

- SPH simulation of a single-binary interaction; courtesy of Dr. Evghenii Gaburov (Northwestern University)

Simulated Three-Body Encounter

- Three-body exchange interaction calculated using the FEWBODY scattering code; courtesy of Dr. Aaron Geller (Northwestern University)

Current Applications

- Dynamical evolution of:
  - Dense stellar systems
    - globular clusters
    - Galactic center
  - Planetary systems (stability?)
  - Orbits of asteroids and comets in the solar system
  - Orbits of satellites within the Earth-Moon system
- Stellar mergers induced by:
  - Kozai oscillations in triple star systems
  - Dynamical encounters involving single, binary and triple stars

⇒ **An exact solution to the general three-body problem would be very useful**

References

- M. Valtonen and H. Karttunen, “**The Three-Body Problem**”, 2006
- D. Heggie and P. Hut, “**The Gravitational Million-Body Problem**”, 2003

- What would be great is if we could somehow find a time-averaged diffusion coefficient for how the total energy for each star should change over the course of a given encounter. This would allow us to calculate how each E_i should change in time, and will ultimately tell us what the outcome will be.
- As our triangles show, there is a symmetry to the problem that can perhaps be exploited.

Adapting the Mean Free Path Formalism

- Encounter rate $\Gamma_{x+y} = N_x n_y \sigma_{x+y} v_{x+y}$, where
  - N_x = Number of single (N_s), binary (N_b) or triple (N_t) stars
    - N = Total number of objects
    - $N_s = (1 - f_b - f_t)N$
    - $N_b = f_b N$
    - $N_t = f_t N$
  - n_y = Number density of single, binary or triple stars
  - σ_{x+y} = Gravitationally-focused cross-section for collision
  - v_{x+y} = Average relative velocity at infinity

References

Andronov, N., Pinsonneault, M. H. & Terndrup, D. M. 2006, AJ, 646, 1160

Chen, X. & Han, Z. 2008a, MNRAS, 384, 1263

Chen, X. & Han, Z. 2008b, MNRAS, 387, 1416

Fan, X., Burstein, D. & Chen, J.-S., et al. 1996, AJ, 112, 628

Fregeau, J. M., Cheung, P., Portegies Zwart, S. F & Rasio, F. A. 2004, MNRAS, 352, 1

Gaburov, E., Lombardi, J. & Portegies Zwart, S. 2010, MNRAS, 402, 105

Hurley, J. R., Pols, O. R., Aarseth, S. J. & Tout, C. A. 2005, MNRAS, 363, 293

Lajoie, C-P. & Sills, A. 2010, ApJ, submitted

Latham, D. W. 2005, Highlights of Astronomy, 14, 444

Leonard, P. J. T. 1989, AJ, 98, 217

Mardling, R. A., 2001, in ASP Conf. Ser. 229, 101

Mathieu, R. D. & Geller, A. M. 2009, Nature, 462, 1032

McCrea, W. H. 1964, MNRAS, 128, 147

Perets, H. B. & Fabrycky, D. C. 2009, ApJ, 697, 1048

Sandquist, E., Latham, D. W., Shetrone, M. D. & Milone, A. A. E. 2003, AJ, 125, 810

Sarajedini, A., et al. 2007, AJ, 133, 1658

Shara, M. M., Saffer, R. A. & Livio, M. 1997, ApJ, 486, L59

Van den Berg, M., Orosz, J., Verbunt, F. & Stassun, K. 2001, A&A, 375, 375