Particle Astrophysics and Cosmology (SS 08)

Homework no. 6 (May 28, 2008)

Tutorials: Wednesday, 17:15 to 18:45, AVZ, room 116 (first floor)

1 Dynamics of Expansion

1.1 Cosmic time as a function of redshift

In case of a flat universe (which we may actually have):

$$\Omega_{m_0} + \Omega_\Lambda = 1$$

Derive the integral that expresses cosmic time as a function of redshift.
(Hint: Write down the Friedmann equation in a convenient form. Then convert $\dot{a}$ into dz/dt. The solution of the resulting integral was presented in the lecture.)

1.2 Cosmological deceleration

An object is observed at redshift z with the density parameter Ω. Calculate the observed rate of change of redshift, dz/dt, for the object. What fractional precision in observed frequency would be needed to detect cosmological deceleration in a decade?
(Hint:

$$\frac{dz}{dt_{obs}} = \frac{\dot{a}_{obs}}{a_{em}} - \frac{\dot{a}_{em} \cdot a_{obs}}{a_{em}^2} \cdot \frac{dt_{em}}{dt_{obs}} \qquad)$$

2 Luminosity distance and *standardarizable* candles

So far we have mainly discussed the scale factor dependence on time for various cosmological models. The observables in astronomy however are the brightness and the redshift of the radiation recorded with telescopes or satellites. Here we are going to derive a relation between the two and compare it to the data from SN1A.

1. Remind yourself of the definition of the luminosity distance d_L and give the relation between the luminosity L of an object in the universe and the flux F recorded with a detector!

2. Under which circumstances would d_L correspond to the proper distance to the luminous object?

3. In realistic models of the universe the flux detected at the time t_0, which was emitted from an object with luminosity L at the time t_e is given by:

$$F = \frac{L}{4\pi a_0^2 r_{\text{com}}^2} \frac{1}{(1+z)^2} \text{ with } r_{\text{com}} = \int_{t_e}^{t_0} \frac{dt}{a(t)} \; .$$

Explain the form of the equation and the individual terms!

We are interested in a relation between the luminosity distance d_L (or the observable flux F) and the observable redshift z. Instead of solving the cosmological evolution exactly, which is only possible numerically in the presence of Ω_Λ, and Ω_m, it is convenient to perform some approximations for the evolution of the scale factor around our current time t_0.

4. Determine the dependence of d_L on q_0, H_0 and z up to terms of the order $\mathcal{O}(z^2)$! Compare your result with the well known Hubble law.

5. Recall the dependence of the deceleration parameter q_0 on the energy densities (cf. previous exercise)! Give a relation when the universe is accelerating and decelerating today!

Astronomers determine the apparent magnitude m and the absolute magnitude M of a galaxy instead of its flux F and luminosity L. They are related as follows:

$$m = A - 2.5 \lg F \quad \text{and} \quad M = A - 2.5 \lg L + 87.45 \; .$$

The peculiar form of this relation arises because the absolute magnitude M is defined as the magnitude the source would have at a distance of 10 pc. Moreover the magnitudes are increasing for fainter objects and there are five steps in magnitude for a range of 100 in brightness. The A is a constant that depends on the part of the spectrum that is used for observation.

6. Derive an expression for the distance modulus $|m - M|$ as a function of the luminosity distance d_L!

7. Sketch the dependence of the distance modulus on the redshift z for an empty universe, a universe with vanishing cosmological constant and the current best fit universe!

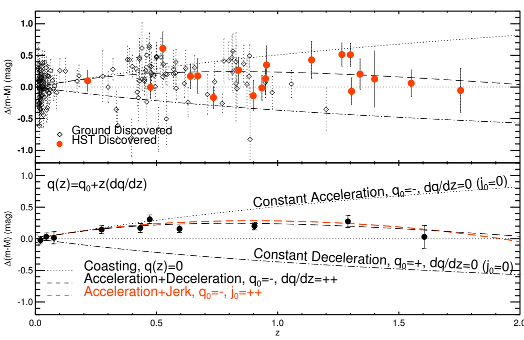

Figure 1: SN 1a data from HST (Riess *et al.*, A.J. 2004)